Accounting for Real Exchange Rate Changes-Extending the Methodology

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Abstract

The seminal work of Engel (1999) establishes a method for measuring the contributions of traded goods and nontraded goods to the fluctuation of real exchange rates. Two difficulties arise: the assumed price aggregation is likely biased, and the needed data on non-traded goods price is often limited. The present work overcomes these difficulties, as follows. First we generalize Engel's exchange rate decomposition to a broader class of price indices, and then we show that data on non-traded prices can be avoided altogether, regardless of which price index is used.

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1 Introduction

The role of nontraded goods in explaining the fluctuations in the real exchange rate (RER) has been widely appreciated in theoretical models.[5] However, in a seminal empirical paper Engel (1999) decomposed the RER for the U.S. vis-a-vis developed countries in terms of traded goods and nontraded goods and found that, contrary to expectations, the role of relative prices of nontraded goods in U.S. real exchange rate fluctuations was insignificant. Parsley (2007) obtained similar results for the real exchange rates of East Asian countries. These results cast doubt on the merit of theoretical models focusing on the presence of the nontraded goods in explaining the fluctuations of the RER (Obstfeld and Rogoff 2000). The presence of highly volatile nominal exchange rates, observed between-country differences in institutions, and difference in data quality may bias the results of such an exercise. Consequently, researchers examined data for the fixed exchange rate/managed floating period and found that nontraded goods do explain some portion of the RER movement (Mendoza 2000).

To shed light on the contribution of nontraded goods to RER changes under permanently fixed nominal exchange rates, Chen, Choi and Devereux (2006) decomposed the U.S. regional real exchange rates following Engel's (1999) methodology, and found the share of nontraded goods (in accounting for real exchange rate fluctuations) to be much higher than that in Engel's between-country study. They argue that the share of nontraded expenditure in Engel's study is too low to fit U.S. regional data, and that a higher share of expenditure on nontraded goods yields a larger contribution of nontraded goods to RER fluctuations. Chen and Devereux (2003) also examined U.S. city real exchange rates and found that the contribution of nontraded goods is around 40% to city RER changes.

[5]Here "non-traded" goods are those goods traded within a country/region but not between countries/regions, and their possible importance lies in the fact that real exchange rates include prices for these goods.

One methodological aspect of Engel (1999) is the representation of the consumer price index as a geometric weighted average of traded and non-traded goods prices indexes. This approach is consistent with a representative consumer having unitary elasticity of subtitution between traded and nontraded goods. As a matter of fact, by examining time series and cross-section data researchers suggest that the unitary elasticity of substitution between traded and nontraded goods is not the norm but the exception[6]. Stockman and Tesar (1995) found that the elasticity of substitution between traded and nontraded goods in consumption is only 0.44. Mendoza (1995) found the elasticity of substitution is 0.74 for OECD countries while Ostry and Reinhart (1992) found that for poor countries it is less than 1. This implies that, in theory, we need to pay attention to the consumption elasticity of substitution.

To conduct such an exercise we need price indices for nontraded goods, but these are generally not readily available, even for developed countries. As a recourse, Engel (1999) relies on the OECD database to compute price indices for nontraded goods (for details, see the appendix A of Engel 1999) . Parsley (2007) attempts to construct nontraded-goods price indices for Hong Kong, Korea, Malaysia, Singapore, Taiwan, Thailand, and the U.S. following Engel's methodology. This construction of nontraded goods price indices entails a sometimes difficult choice about what should be included among nontraded goods.

The lack of information about nontraded goods prices – including service prices and also rental prices in developing countries – remains a bottleneck for applying Engel's methodology to developing countries. For many developing countries we can obtain a food price index, a clothing price index, and other traded goods price components but still we can't adopt Engel's methodology due to a lack of non-traded goods prices.

We thus identify the following important issues related to adopting Engel's methodology:

[6]A number of papers discuss the role of elasticity of substitution in production in real exchange rate dynamics and find that the elasticity of substitution in production is a crucvial varviable. See Morshed and Turnovsky (2006) for a recent contribution.

(1) the role of the size of the share of nontraded goods in the consumption basket, (2) the consumption elasticity of substitution between traded and nontraded goods, (3) the requirement of additional assumptions concerning the construction of nontraded goods price index, and (4) the measurement issues related to construction of shares and the prices of nontraded goods.

In this paper we show that the data requirements of Engel's methodology are much less demanding than they appear. The methodology can be easily adopted for any country as long as we have a consumer price index and a traded goods price index. The construction of nontraded goods price indices is actually redundant to Engel's methods. In addition, the size of the share of nontraded goods is redundant and thus the Chen et al (2006) criticism about a low share of nontraded goods in Engel's results seems inapplicable. Measurement issues become less severe as we don't need to construct shares or prices of nontraded goods in order to adopt the Engel (1999) methodology. It is also interesting to note that Engel's (1999) results are more general than they appear as far as the issue of consumption elasticity of substitution is concerned. Engel (1999) conducted an exchange rate decomposition compatible with a Cobb-Douglas utility function, but we show that even if we use the more general CES utility functions, or others which are similarly homogeneous (of degree 1), the results will not be different.

The remainder of the paper is organized as follows. Section (2) develops a generalization of Engel's (1999) exchange rate decomposition, Section (3) discusses a problem of identification of exchange rate components, and proposes a solution. Section (4) concludes, and an Appendix contains proofs of some mathematical results.

2 Real Exchange Rate Movements

The real exchange rate (RER) is defined as:

$$Q = \frac{SP^*}{P} \tag{1}$$

with S the nominal exchange rate, P the price level in the home country, and P^* the price level in the foreign country. In a world with parity in purchasing power, Q equals 1, and any movements in the real exchange rate over time imply price disparity at some dates.

In what follows, we first briefly recount Engel's (1999) decomposition of real exchange rate movements, then consider an alternative decomposition which has more straightforward application when the price level is measured by a conventional consumer price index (CPI). We then tie the two cases cases together, and provide a general decomposition that retains the spirit of Engel's approach.

2.1 Engel's Decomposition

The real exchange rate, expressed in logarithms, is:

$$q = s + p^* - p \tag{2}$$

Suppose that there is a log-linear relationship between overall log-price p and that of the traded and non-traded components in the home country:[7]

[7]Equivalently, the price level P can be written as a Cobb-Douglas function $P = (P^T)^{1-\alpha}(P^N)^{\alpha}$ of traded and non-traded price levels P^T and P^N. For this the standard motivation is that markets are in competitive equilibrium, with a representative household that chooses a combination of traded and non-traded goods so as to maximize Cobb-Douglas utility $U(C_T, C_N) = C_T^{1-\alpha}C_N^{\alpha}$ subject to the budget constraint $P^T C_T + P^N C_N = Y$, where Y is household income. The resulting expenditure function is then $Y(P^T)^{1-\alpha}(P^N)^{\alpha}$, which is Cobb-Douglas in price levels.

$$p = (1-\alpha)p^T + \alpha p^N \tag{3}$$

with weight α on the nontraded good. Analogously, for the foreign country suppose that:

$$p^* = (1-\beta)p^{T*} + \beta p^{N*} \tag{4}$$

To describe the influence of nontraded goods on the real exchange rate q, Engel writes q as the sum of two parts, as follows:

$$q = x + y \tag{5}$$

$$x = s + p^{T*} - p^T \tag{6}$$

$$y = \beta(p^{N*} - p^{T*}) - \alpha(p^N - p^T) \tag{7}$$

The RER component x is a log form of real exchange rate specific to traded goods, measuring the disparity between prices of traded goods in the home and foreign country. The component y is more complex, involving parameters α and β that can represent preference for nontraded goods in the relevant economic model.[8] In general, y reflects within-country differences between traded and nontraded goods prices, increasing in the foreign country difference and decreasing in the home country difference.

Engel (1999) and other researchers examined the contributions of x and y to the fluctuations of q and found that the fluctuation in q is mainly generated by fluctuation in x.

[8] See footnote 4 for discussion.

2.2 CPI and Linear Price Indices

Engel (1999) relies on log-linear price aggregation (3)-(4) to decompose real exchange rates into a purely traded-good component and a mixed traded/nontraded component. When applied to consumer price indices, this nonlinear aggregation is worrisome since CPI aggregation is typically linear. If the nonlinear form is wrong, as seems likely, then conclusions about real exchange rates based on Engel's decomposition (5)-(7) are in doubt. To investigate, consider the linear specification of prices P and P^* in the home country and foreign country:

$$P = (1-\alpha)P^T + \alpha P^N \tag{8}$$

$$P^* = (1-\beta)P^{T*} + \beta P^{N*} \tag{9}$$

The real exchange rate Q, defined via (1), is nonlinear in price levels P and P^*, and it is not obvious whether Q factors neatly into a traded-component and a traded vs. nontraded component under the auspices of linear aggregation (8)-(9). However, in light of (2) one can always write the log real exchange rate as follows:

$$\begin{aligned} q &= s + p^{T*} - p^T - (p^{T*} - p^T) + p^* - p && (10) \\ &= x + y && (11) \end{aligned}$$

with traded goods real exchange rate $x = s + p^{T*} - p^T$ as defined earlier and:

$$y = -(p^{T*} - p^T) + p^* - p \tag{12}$$

in which case, applying linear aggregation (8)-(9) and rearranging terms, we have:

$$y = \log\left(\frac{1+\beta\left(\frac{P^{N*}}{P^{T*}}-1\right)}{1+\alpha\left(\frac{P^{N}}{P^{T}}-1\right)}\right) \tag{13}$$

Conceptually, the real exchange rate model based on linear price aggregation (8)-(9) is the same as Engel's model based on log-linear aggregation (3)-(4) . In each case there are two components of the log exchange rate q, these being the traded-goods log real exchange rate x and a term y that is a non-linear function of the relative prices P^N/P^T and P^{N*}/P^{T*} of the nontraded good within each country, with y increasing in foreign country relative price (of nontraded goods) and decreasing in home country relative price. With linear aggregation the mathematical form of y is actually more complicated in terms of the underlying prices, but this does not change its basic interpretation.[9]

2.3 General Decomposition

As a matter of economic curiosity, the fact that two different aggregation methods yield the same type of exchange rate decomposition prompts the question of whether other methods can do the same. The answer is "yes," as we now show.

Consider the constant elasticity of substitution (CES) form of price aggregates:

[9] The two mathematical forms of y are distinct yet also clearly related. Specifically, when P^N is close to P^T and likewise P^{N*} is close to P^{T*}, the first-order (Taylor series) approximation of y in (13) is Engel's formula (7). Hence, if the units of quantity for traded and nontraded goods can be selected so that the price ratios P^N/P^T and P^{N*}/P^{T*} have values close to 1 most of the time, the issue of linear vs. non-linear aggregation may be unimportant. Whether or not this can be achieved is unclear a priori.

$$P = \left((1-\alpha)(P^T)^\theta + \alpha(P^N)^\theta\right)^{1/\theta} \tag{14}$$

$$P^* = \left((1-\beta)(P^{T*})^\theta + \beta(P^{N*})^\theta\right)^{1/\theta} \tag{15}$$

for domestic price level P and foreign price level P^*, with parameter $\theta \neq 1$. For the unit elasticity case ($\theta = 1$), CES takes the Cobb-Douglas (log-linear) form.

Applying (12), we have:

$$\begin{aligned} y &= p^* - p - (p^{T*} - p^T) && (16) \\ &= \log(P^*) - \log(P) - (p^{T*} - p^T) && (17) \\ &= \log(P^*/P) - (p^{T*} - p^T) && (18) \end{aligned}$$

With the CES form for P and P^*, (18) yields:

$$\begin{aligned} y &= \log\left(\frac{\left((1-\beta)(P^{T*})^\theta + \beta(P^{N*})^\theta\right)^{1/\theta}}{\left((1-\alpha)(P^T)^\theta + \alpha(P^N)^\theta\right)^{1/\theta}}\right) - (p^{T*} - p^T) && (19) \\ &= \log\left(\frac{\left((1-\beta)(P^{T*})^\theta + \beta(P^{N*})^\theta\right)^{1/\theta}}{\left((1-\alpha)(P^T)^\theta + \alpha(P^N)^\theta\right)^{1/\theta}} \frac{P^T}{P^{T*}}\right) && (20) \end{aligned}$$

Simplifying (20), and rearranging terms, provides:

$$y = \frac{1}{\theta}\log\left(\frac{1 + \beta\left(\left(\frac{P^{N*}}{P^{T*}}\right)^\theta - 1\right)}{1 + \alpha\left(\left(\frac{P^N}{P^T}\right)^\theta - 1\right)}\right) \tag{21}$$

The interpretation is the same as before, with y a non-linear function of the nontraded goods (relative) price within each country, increasing in foreign country relative price and decreasing in home country relative price.[10]

The CES aggregation form contains the Cobb-Douglas and linear forms considered earlier, but does not exhaust the possibilities for getting a real exchange rate decomposition in the general spirit of Engel (1999). To consider other possibilities, write y as a general function g of its arguments:

$$y = g(P^T, P^N, P^{T*}, P^{N*}) \tag{22}$$

In the CES case, g is homogeneous of degree zero with respect to (positive) scalar multiplication of its first two arguments, and similarly homogeneous with respect to its last two arguments:

$$g(\gamma P^T, \gamma P^N, \delta P^{T*}, \delta P^{N*}) = g(P^T, P^N, P^{T*}, P^{N*}) \tag{23}$$

for all positive numbers γ and δ.[11] Any weighted average of such homogeneous functions is again homogeneous in the same sense, [12].

Considering the price aggregator f as an input, and the y-defining function g as an output, if we ask that g be homogeneous in the sense of (23) then we can look for those inputs f that deliver this homogeneity. To this end we have the following:

[10]Also, when the relative prices P^N/P^T and P^{N*}/P^{T*} are close to 1, the first-order (Taylor series) approximation of y in (21) is Engel's formula (7), a simplification we observed earlier in the case of linear aggregation.

[11]Recall that a function $h(z_1, z_2)$ is homogeneous of degree zero for all positive numbers z_1, z_2 if and only if it can be rewritten as $h(z_1, z_2) = q(z_1/z_2)$ for some function q. See for example Hazewinkel, Michiel (2001).

[12]That is, the set of functions g satisfying (23) is a convex set.

Proposition **1** *Given the exchange rate accounting identities (10)-(12), let the general price level be* $P = f(P^T, P^N)$ *in the home country and* $P^* = f(P^{T*}, P^{N*})$ *in the foreign country, for some differentiable function* f*, and let the exchange rate component* y *take the form* $y = g(P^T, P^N, P^{T*}, P^{N*})$ *for some differentiable function* g*. Then* g *is homogeneous of degree 0 with respect to* (P^N, P^T) *and to* (P^{N*}, P^{T*}) *if and only if the price aggregator* f *is homogeneous of degree 1.*

In light of Proposition 1, to get an exchange rate component y with the desired homogeneity (23) we can and must choose a price aggregator f which is homogeneous of degree 1.[13] The CES form of f is an example but so too is any weighted average of CES forms.

We should also insist that the price aggregator f be increasing in its arguments, in which case the effect on the y-defining function g is as follows:

Proposition **2** *Suppose that the price aggregator function* f *is differentiable and homogeneous of degree 1. Then* f *is a strictly increasing function if and only if the exchange rate component function* g *is strictly increasing in* P^T *and* P^{N*} *while strictly decreasing in* P^N *and* P^{T*}*.*

3 Identification

To apply Engel's exchange rate decomposition, or the alternative decompositions discussed in Sections 2.2 and 2.3, we need to measure the components x and y of the (log) real exchange rate.

[13]Here we are assuming – as in Proposition 1 – that the same price aggregator f is used to get a price index in the home and foreign country. An extension of the Proposition to allow country-specific fs would be interesting but is beyond the present scope.

3.1 Engel's identification method

Engel tackles the identification problem in two steps. First he estimates the parameters α and β appearing in the log-linear aggregation rule (3)-(4), then he plugs these parameter estimates into the formula (7) to get an estimate of the RER component y. To this end he disaggregates the price level across several categories of "traded" goods, e.g. commodities, and "nontraded" goods, e.g. services. Applying log-linear aggregation across the various goods and services, he estimates the relevant weights via ordinary least squares (OLS) regression of the (time-differenced, log) aggregate price level on the price levels of the goods and services. With the weight estimates he aggregates across commodities to get an estimate of (time-differenced) values of both components x and y of the (log) real exchange rate q.

We will examine Engel's identification method more closely, and to this end we briefly state the relevant technical details. Given an aggregate price index P_t observed over time, and a set of disaggregated price indices P_{kt}, across $k = 1, 2, ..., K$ categories of goods and services, log-linear aggregation takes the form:

$$P_t = \prod_{k=1}^{K} P_{kt}^{\gamma_k} \tag{24}$$

for some constants $\gamma_1, ..., \gamma_K$ that sum to 1:

$$\sum_{k=1}^{K} \gamma_k = 1 \tag{25}$$

Engel (1999) estimates the coefficient vector γ via the regression model:

$$\Delta p_t = \sum_{k=1}^{K} \gamma_k \Delta p_{kt} + u_t \tag{26}$$

with regression error u_t, via OLS subject to the constraint (25), this being equivalent to the unconstrained OLS regression:

$$\Delta p_t - \Delta p_{tK} = \sum_{k=1}^{K-1} \gamma_k(\Delta p_{it} - \Delta p_{Kt}) + \varepsilon_t \tag{27}$$

With OLS coefficient estimates $\hat{\gamma}$ in hand, Engel aggregates to get proxies for commodity and service (differenced log) price levels:

$$\Delta p_t^T = \sum_{k \in T} \hat{\gamma}_k \Delta p_{it}, \quad \Delta p_t^N = \sum_{k \in N} \hat{\gamma}_k \Delta p_{it}, \tag{28}$$

as well as weights α and β for these spending categories:

$$\alpha = \sum_{k \in T} \hat{\gamma}_k, \quad \beta = \sum_{k \in N} \hat{\gamma}_k, \tag{29}$$

Engel (1999) reports the CPI component weights α and β for various countries, but not the underlying regression coefficients $\hat{\gamma}$. Using his CPI data for six countries[14], we re-run his regressions and report the coefficients in Table 1. In the table the weights α and β – which sum to 1 – appear as reported values for Commodities and for Services, respectively. Within each of the spending categories, the sub-category weights are regression coefficients $\hat{\gamma}$.

A problem, apparent from Table 1, is that the CPI component weights obtained via OLS regression can be negative, as is the case for the U.S. Rent component of Services. The true weight is positive, and while a negative estimate is not damning it is sure to raise eyebrows among some economists. Engel (1999) himself does not comment on the issue, but perhaps adapted to it somehow. His reported values of α and β coincide with those in our Table 1 except in the problematic case of the U.S., where his values are $\alpha = 0.541$ and $\beta = 0.459$. Assuming that his disclosed data is the same data used in his work, his reported results to do not completely follow his stated methodology, and instead deviate from this methodology

[14]The countries are Canada, France, Germany, Italy, Japan, and the U.S.

in the one instance where it proves problematic.

3.2 General identification

Earlier we generalized Engel's (1999) exchange rate decomposition, and to apply this more general exchange rate model we must somehow identify its components x and y. One possibility is to extend Engel's regression approach, first estimating the relevant parameters and then using the parameter estimates to construct CPI component series and the real exchange rate components x and y. In the setting of the more general CES decomposition in Section 2.3, the idea would be to assume a CES form of price aggregation across all goods categories, then transform prices so that the relevant parameters (α, β, θ) can be expressed in terms of coefficients in a regression of transformed prices. However, while Engel's regressions involve inflation rates – in the form of differenced log price levels, these same regressions do not generally identify the CES parameters.

Even if we could find a way to transform prices so that a price regression could deliver estimates of the exchange rate components x and y, there remains the problem shown earlier of possibly negative weight estimates for some spending categories. A simple solution is to avoid weight estimation altogether, and rely on officially reported price indices for all goods and services, and for commodities. With these two indices in hand, their time-differenced log values coincide with the variables Δp and Δp^T respectively. Assuming that the exchange rate (Q) is also available, this turns out to be sufficient information to compute exchange rate components x and y. To see why, note first that from the definition (6), the differenced traded goods RER component is:

$$\Delta x_t = \Delta q_t + \Delta p_t^{T*} - \Delta p_t^T \tag{30}$$

hence is computable from the log-exchange rate q and the commodities CPI indices in the home and foreign countries. As for the remaining RER component, note from the accounting identities (10)-(13) that $q = x + y$ and so we can write:

$$\Delta y_t = \Delta q_t - \Delta x_t \tag{31}$$

hence the value of Δy_t is trivially available once the exchange rates and traded goods price indices are in hand.

With the proposed identification of y in terms of p and x, the practitioner can then compute the sorts of statistics in Engel (1999), including ratios of mean squared errors (MSE) such as:

$$\frac{MSE\ (x_t - x_{t-n})}{MSE\ (x_t - x_{t-n}) + MSE\ (y_t - y_{t-n})} \tag{32}$$

with MSE defined as variance plus squared mean, each estimated on the relevant sample.

4 Conclusions

Engel's seminal paper (Engel 1999) on the decomposition of real exchange rates – into traded and nontraded components – has received a huge response. A Google search of this paper in October 2012 yields more than 650 citations. Engel's methodology has been used to conduct similar decomposition exercises for a wide variety of country groups and even for regions within a country. In this paper we show that Engel's methodology is more general, and the data and information requirements are fewer, than what the literature suggests. We also identify some potential problems related to the identification of traded and nontraded components in Engel's approach, and we provide some suggestions for dealing with those.

Future application's of Engel's decomposition should include more studies of developing countries, and while data requirements make this task appear daunting, our results show that such applications are feasible. Too, our finding that the decomposition is invariant to the method of price aggregation – so long as it is compatible with constant elasticity of substitution (CES) – adds some robustness that may be particularly useful in application to developing countries.

References

Chen, L., S. Choi, and J. Devereux (2006). "Accounting for U.S. Regional Real Exchange Rates," Journal of Money, Credit, and Banking, 38, 229-244.

Chen, L. and J. Devereux (2003). "What Can US City Price Data Tell Us About Purchasing Power Parity?" Journal of International Money and Finance, 22, 213-222.

Engel, C. (1999). "Accounting for U.S. Real Exchange Rate Changes," Journal of Political Economy, 107, 507-538.

Hazewinkel, Michiel (2001). "Homogeneous Function," Encyclopedia of Mathematics, Springer.

Mendoza, E. (1995). "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations," International Economic Review, 36, 101-137.

Mendoza, E. (2000). "On the Instability of Variance Decompositions of the Real Exchange Rate across Exchange-Rate-Regimes : Evidence from Mexico and the United States," NBER Working Paper No. 7768.

Morshed, A.K.M. M., and S. J. Turnovsky (2006)."Elasticity of Substitution and the Persistence of the Deviation of the Real Exchange Rates." Review of Development Economics, 10, 411-433.

Obstfeld, M., and K. Rogoff (2000). "The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?" in: Bernanke, B. and K. Rogoff (Eds.), NBER Macroeconomics Annual, 15, 339-390.

Ostry J. and C. Reinhart (1992). "Private Saving and Terms of Trade Shocks: Evidence from Developing Countries," IMF Staff Papers 39, 495-517.

Parsley, D. (2007). "Accounting For Real Exchange Rates in East Asia," Journal of International Money and Finance, 26, 1355-1377.

Stockman, A. and L. Tesar (1995). "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," American Economic Review, 85, 168-185.

Appendix

PROOF OF PROPOSITION 1

For the "if" part of this if-and-only-if proposition, suppose that g is homogeneous in the sense of (23) in the text then, with y of the form (13), we can write it as:

$$y = \log(f(\delta P^{T*}, \delta P^{N*}) - \log(f(\gamma P^T, \gamma P^N)) - \big(\log(\delta P^{T*}) - \log(\gamma P^{N*})\big) \tag{A-1}$$

for each positive γ and δ. Expanding terms then yields:

$$y = \log(f(\delta P^{T*}, \delta P^{N*})) - \log(f(\gamma P^T, \gamma P^N)) - (p^{T*} - p^{N*}) - (\log(\delta) - \log(\gamma)) \tag{A-2}$$

Differentiating y in (A-2) with respect to γ, at $\gamma = 1$, and setting the derivative equal to zero (as it must be), yields:

$$f(P^T, P^N) = f_1(P^T, P^N)P^T + f_2(P^T, P^N)P^N \tag{A-3}$$

with the same result when differentiating with respect to δ at $\delta = 1$. Applying Euler's (homogeneous function) theorem, we find that f is homogeneous of degree 1.

For the "only-if" part of of the proposition, suppose that f is homogeneous of degree 1. Then we can write:

$$f(\gamma P^T, \gamma P^N) = \gamma f(P^T, P^N) \tag{A-4}$$

$$f(\delta P^{T*}, \delta P^{N*}) = \delta f(P^{T*}, P^{N*}) \tag{A-5}$$

in which case, from (A-2), y is homogeneous in the sense of (23). $\blacksquare$

PROOF OF PROPOSITION 2

Applying (A-2), with f increasing and differentiable note that:

$$\frac{\partial y}{\partial P^{N*}} = f_2 > 0 \tag{A-6}$$
$$\frac{\partial y}{\partial P^{N}} = -f_2 < 0 \tag{A-7}$$

Moreover, with g homogeneous in the sense of (23), by Euler's theorem we have:

$$g_1 P^T + g_2 P^N = 0 \tag{A-8}$$
$$g_3 P^T + g_4 P^N = 0 \tag{A-9}$$
$$\tag{A-10}$$

in which case g_1 and g_2 are of opposite signs, as are g_3 and g_4, so:

$$\frac{\partial y}{\partial P^{T*}} < 0 \tag{A-11}$$
$$\frac{\partial y}{\partial P^{N*}} > 0 \tag{A-12}$$

as was to be shown. $\blacksquare$

Table 1: Estimated weights for spending categories, Engel data

Sector	Canada	France	Germany	Italy	Japan	USA
Commodities	0.606	0.747	0.719	0.756	0.689	0.619
Food	0.245	0.328	0.298	0.442	0.337	0.249
Goods Less Food	0.360	0.419	0.421	0.314	0.351	0.370
Services	0.394	0.253	0.281	0.244	0.311	0.381
Rent	0.130	0.111	0.138	0.056	0.170	-0.004
Services Less Rent	0.264	0.142	0.143	0.187	0.141	0.385