Taxes and the present value assessment of economic losses in personal injury litigation: Comment[1]

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Abstract

Anderson and Barber (2010) provide a recent discussion of tax effects on economic damages, for forensic economists and similar experts who supply the courts with opinions on economic damages. Anderson and Barbers' paper fills a void in the forensic economics literature, by offering a formal theory of how tax considerations can impact economic damages. In the present work I point out a limitation of this theory - via a counter-example, and discuss conditions under which which the theory seems to hold approximately.

Keywords: income stream; present value; tax; personal injury; wrongful death

I. Introduction

In court cases involving personal injury and wrongful death, economic damages typically the present value of future incomes which would have been earned – if not for the injury or death. For the typical U.S. worker, it is natural for their income to grow over most of their worklife due to inflation and increases in productivity. When valuing a growing income stream, the forensic economist (FE) has various factors to consider, including the inclusion or exclusion of tax -- in the worker's wages and also in interest income associated with the investment fund required to replicate lost future wages. The FE determines whether or not to include tax or not depending on legal requirements of the court in which the case is filed, and on the FE's opinion about the reasonableness of pre-tax vs. post-tax values. In cases where the treatment of tax is subject to the FE's choice, it is important to be able to explain the consequences of such choices to the court. Does the inclusion of tax tend to raise economic damage estimates, or not? Economic theory is a potentially valuable resource for forensic economists who wish to be able to answer this question confidently at trial and in deposition.

In terms of economic theory, the effect of tax on the present value of a growing income stream has not been addressed in the general economics literature. Over the last 25 years forensic economists have published research in journals specialized to their field, including the Journal of Forensic Economics and the Journal of Legal Economics. In both journals, a number of research papers have discussed tax and present value. In the Journal of Forensic Economics, this includes some early work by Goodwin and Paul (1988) and Ciecka (1989), and a special issue in 1994 (volume 7, number 3) devoted to the subject of tax and economic awards. In the Journal of Legal Economics, the recent work of Gary Anderson and Joel Barber (2010) makes a number of contributions to the literature, including a discussion of tax effects on the present value of services and medical care. It also surveys the existing literature, and posits a theoretical relationship between tax and the present value of growing income.

Anderson and Barber (2010) posit that tax has a negative effect on the present value of growing income when the number future pay periods is small, and has a positive effect when the number of periods is large. They also provide a mathematical formula for the “breakeven” point -- the number of periods at which the tax effect switches from negative to positive. This theoretical work is insightful and extends the scope of the forensic economics literature.

The present work reconsiders the effect of tax on the present value of growing income. I show by counter-example that the Anderson and Barber (2010) tax theory does not generally hold in the mathematical form in which they state it, and I identify the source of error. I also provide examples where their breakeven formula is nearly correct, giving some hints for future research on the effects of tax on economic damages.

II. Anderson and Barber (2010)

Anderson and Barber (2010), henceforth abbreviated *AB*, analyze the effect of tax on the present value of earnings, under the assumption that earnings grow at a constant rate g over time:

$$E_t = E_0(1+g)^t \tag{2.1}$$

with E_0 the base earnings earned in period 0. Anderson and Barber posit an interesting theory about how tax affects the present value of earnings. I briefly summarize their results in Section II.A, then examine them in more detail in Sections II.B-II.C.

A. Tax Effect

Anderson and Barber (2010) use the following notation and formulas for the present value of earnings, with and without tax:

(2.2)
$$PV_{earningsaftertax} = \frac{E_0(1-\tau^e)(1+g)}{(r(1-\tau^i)-g)}\left[1-\left(\frac{1+g}{1+r(1-\tau^i)}\right)^N\right]$$

(2.3)
$$PV_{earningswithouttax} = \frac{E_0(1+g)}{(r-g)}\left[1-\left(\frac{1+g}{1+r}\right)^N\right]$$

where r is the interest rate on a (riskless) bond, τ^e is the tax rate on earnings, and τ^i is the tax rate on interest.

In terms of tax effects, *AB* posit a breakeven point, such that tax has a negative effect on present value when the number of earnings periods is less than some value which I will call N^*, and has a positive effect on present value when the number of periods exceeds N^*. In other words, they posit that $PV_{earningsaftertax}$ is smaller than $PV_{earningswithouttax}$ when N is small, and that the reverse relationship holds when N is large. They identify the breakeven point via the following equation:

(2.4)
$$\tau^e = -\frac{\tau^i rD}{(1+r)}$$

where D is the constant:

(2.5) $$D = -\frac{(1+r)}{(r-g)} + \frac{N\left(\frac{1+g}{1+r}\right)^N}{\left[1-\left(\frac{1+g}{1+r}\right)^N\right]}$$

According to *AB*, for smaller N the left-hand side of (2.4) is larger than the right-hand side, and the after-tax present value of earnings is less than the without-tax present value. For larger N, the left-hand side is smaller than the right-hand side, and the after-tax present value of earnings is greater than the without-tax present value.

B. (Counter)Example

Anderson and Barber provide an example to illustrate their mathematical theory. They set the tax rate on earnings equal to that on interest, with $\tau^e = \tau^i = 0.1$, and they set the growth rate on earnings equal to the before-tax rate of interest, with $g = r = 0.05$. They find that the breakeven condition (2.4) holds with $D = -21$ and that this value of D is achieved at $N = 40$. They conclude that, for the given values of g, r, τ^e and τ^i, adjustment for tax will result in a decrease in the present value of loss with the introduction of taxes in situations where $N < 40$, and will increase the present value of loss in situations where $N > 40$.

Examining the present value formulas (2.2)-(2.3) and the definition (2.5) of the constant D, some trouble arises. Since the difference $r - g$ equals 0, the without-tax present value (2.3) and the formula (2.5) for D are undefined. Neither can be a finite quantity, as both involve division by zero. Also, they can not be reliably assigned the value plus infinity (+∞) or minus infinity (-∞), as both values emerge in the limit as we let $r - g$ converge to zero from above or, alternatively, from below. Consequently, it is not possible to check the breakeven condition (2.4) in Example 1 via these formulas, or to apply this condition to the determination of tax effects.

While formulas and (2.3) and (2.5) do not apply in this example, Anderson and Barber provide alternative formulas that do, these being:

(2.6)
$$PV_{earningswithouttax} = \sum_{t=1}^{N} \frac{E_0(1+g)^i}{(1+r)^i}$$

and:

(2.7)
$$D = -\frac{\sum_{t=1}^{N} \frac{tE_0(1+g)^t}{(1+r)^t}}{\sum_{t=1}^{N} \frac{E_0(1+g)^t}{(1+r)^t}}$$

which are well-defined for any positive values of g, r, and E_0. To check the posited value -21 for D, I apply the *AB* breakeven condition (2.4) to get:

(2.8)
$$D = \frac{-(1+r)\tau^e}{r\tau^i} = -21$$

which matches their result.

We can also check the posited value 40 for the number N of future work periods. Given the value for D, I solve for N by applying the formula (2.7) with total offset ($g = r$):

(2.9) $$D = -\frac{\sum_{t=1}^{N} t}{\sum_{t=1}^{N} 1}$$

(2.10) $$= -\frac{N(N+1)/2}{N}$$

(2.11) $$= -\frac{N+1}{2}$$

The value N^* of N that satisfies the breakeven condition (2.4) when $\tau^e = \tau^i$ is then:

(2.12) $$N^* = 1 + \frac{2}{r}$$

With $r = 0.05$, (2.12) yields $N^* = 41$, which is nearly the same as the value 40 mentioned by Anderson and Barber.

Turning now to the posited tax effects, I compute present values and report them in Table 1, with base earnings E_0 set equal to 1. At each horizon N, present value without tax equals N due to total offset. At $N = 39$, PV without tax is larger than PV with tax, hence tax lowers present value. The same is true at $N = 40$ through 42, whereas Anderson and Barber suggest that for N larger than 40 (or 41) tax should raise present value. At $N = 43$ tax raises PV, hence if there is a breakeven value of N at which the tax effect goes from negative to positive, it is 42 or 43, which is inconsistent with *AB*'s breakeven condition (2.4).

C. Other Evidence

To further examine the workability of the breakeven condition (2.4) for tax effects, consider some more examples. As earlier, let there be total offset between the growth rate on earnings and the interest rate, with $g = r = 0.05$, and let the tax rate on earnings be equal to the tax rate on interest. The breakeven value N^* of the earnings horizon N, which I derived earlier, is again $N^* = 41$ here because r has not changed.

Table 2 reports present values with and without tax, analogous to Table 1, but with a lower tax rate: $\tau = 0.01$. As shown, tax lowers present value at $N = 39, 40, 41$, and raises it at $N = 42, 43, 44$, which is consistent with Anderson and Barbers' posited tax effects. A possible explanation is that their theory is applicable when tax rates are sufficiently small, but not for all tax rates. As a further check, I report in Table 3 results for a high tax rate: $\tau = 0.5$. Here the discrepancy between posited tax effects and actual effects is dramatic. Tax lowers present values at $N = 39,, 40, ..., 51$, and raises present values for $N = 52, 53, 54$, hence if there is a breakeven value of N it must be between 51 and 52, rather than the value $N^* = 41$.

Based on the foregoing examples, a possibility is that there is a breakeven value for N which increases with the tax rate τ. In support of this idea, Table 4 reports breakeven possibilities for N at the tax rates $\tau = 0.01, 0.05, 0.1, 0.2, ..., 0.5$. At each stated value of N, tax lowers present value for nearby smaller values of N, and raises present value for nearby larger values of N. As indicated, these candidate breakeven values are increasing in the tax rate.

III. Forensics

To explain the pattern of observed discrepancies between actual tax effects and those postulated by Anderson and Barber (2010), let's consider the arguments underlying them. The crux of the matter is the breakeven condition (2.4), which for *AB* represents a balancing of offsetting effects: on the left-hand side of (2.4) is a loss of present value associated with tax on income, while on the right-hand side is a gain in present value associated with tax on interest. The validity of this balancing act is not obvious, and involves some reasoning in terms of elasticities and also the constant D.

Anderson and Barber refer to D as duration, but if duration refers to time then it should be positive, or at least non-negative, whereas Anderson and Barbers' D is always negative. As a first step in reconsidering the breakpoint condition (2.4) I discuss the concept of duration, then turn to elasticities and the marginal analysis of offsetting tax effects.

A. Duration

Anderson and Barber identify the constant D as *duration* in the sense of Macaulay (1938). To get a sense of what Macaulay means by duration in this work, here is a quote -- Macaulay (1938, Chapter 2, page 48):

Now, if present value weighting be used, the 'duration' of a bond is an average of the durations of the separate single payment loans into which the bond may be broken up. To calculate this average the duration of each individual single payment loan must be weighted in proportion to the size of the individual loan; in other words, by the ratio of the present value of the individual future payment to the sum of all the present values, which is, of course, the price paid for the bond.

For a riskless bond paying a coupon amount I in future periods 1, 2,..., N, and returning a face value F in period N, with gross yield $R=1+r$, Macaulay computes his duration D as follows:

(3.1)
$$D = \frac{\dfrac{I}{R} + \dfrac{2I}{R^2} + \cdots + \dfrac{NI}{R^N} + \dfrac{NF}{R^N}}{\dfrac{I}{R} + \dfrac{I}{R^2} + \cdots + \dfrac{I}{R^N} + \dfrac{F}{R^N}}$$

This D is a weighted average of the dates 1, 2,...., N, and as such its value lies between the values 1 and N, a positive value.

While Macaulay (1938) focuses on bonds, his concept of duration -- as a present-value weighted average of payment dates -- is applicable to any known positive income stream over future dates $t = 1, 2, ..., N$:

(3.2)
$$D = \sum_{t=1}^{N} t \frac{PV_t}{PV}$$

with total present value PV and PV_t the present value of the earnings arriving in year t. For additional discussion of Macaulay's duration, see Weil (1973).

Applied to the bond example (3.1), the general duration formula (3.2) has the following components:

(3.3)
$$PV = \frac{I}{R} + \frac{I}{R^2} + \cdots + \frac{I}{R^N} + \frac{F}{R^N}$$

(3.4)
$$PV_1 = \frac{I}{R}$$

(3.5)
$$PV_2 = \frac{I}{R^2}$$

and so on, until:

(3.6)
$$PV_{N-1} = \frac{I}{R^{N-1}}$$

(3.7) $$PV_N = \frac{I+F}{R^N}$$

For a general earnings stream $E_1,...,E_N$, the (pre-tax) present value of income arriving in future period t is:

(3.8) $$PV_t = \frac{E_t}{(1+r)^t}$$

and Macaulay's duration is:

(3.9) $$D = \frac{\sum_{t=1}^{N} \frac{tE_t}{(1+r)^t}}{\sum_{t=1}^{N} \frac{E_t}{(1+r)^t}}$$

Macaulay (1938, page 51) illustrates bond duration by computing it for a variety of bonds with different yields and years to maturity. In Table 5 I report duration for growing income streams, with D computed via formula (3.9) and earnings growth rate $g = 0.05$. As in Macaulay's work, all duration values are positive. In the last row of Table 5 I report results in the limit case where horizon N approaches infinity. For the cases $g \geq r$ it is easy to show that $D \uparrow \infty$ as $N \uparrow \infty$. For the case $g < r$ I use Anderson and Barbers' D formula (2.5), multiplied times -1.

John Hicks (1939, page 186) independently develops the duration formula (3.9), which he calls the "average period" of the income stream. Again, duration is a measure of time, hence is non-negative. Popular software for financial calculations, including Microsoft Excel 2010 and Wolfram Mathematica 8, implement duration in these terms. For example, consider a 5 year bond that pays no coupon and has an interest rate of 10 percent. Its only payment is at the end of 5 years, so Macaulay's duration is $D = 5$ in accordance with the following Excel function: =duration(01/01/10,1/1/2015,0,0.1,1).

Could the errors generated by Anderson and Barbers' postulated tax effects be due to a wrong sign -- negative rather than positive -- for their Macaulay duration formula? The answer turns out to be “no,” the reason being that if we replace Anderson and Barbers' constant D with its negative $-D$ in the breakeven formula (2.4), we get a new formula which can't work unless the negative sign on the right-hand side is itself reversed -- otherwise the positive value on the left-hand side can not match a positive sign on the right-hand side -- which then reverts to the original formula. Despite this fact, it is helpful to have some understanding of Macaulay's duration, as it plays an integral part in Anderson and Barbers' basic argument motivating the breakeven condition (2.4). This argument involves elasticities, and as shown in the next section the argument is not quite right. It does however contain useful elements, and to appreciate these fully it will be helpful to note here that, according to Hicks (1939), the duration formula (3.9) can also be interpreted as the elasticity of present value with respect to the “discount factor” $1/R$. That is, with present value expressed as:

(3.10)
$$PV = \sum_{t=1}^{N} R^{-t} E_t$$

and its elasticity defined as:

(3.11)
$$\frac{\partial PV}{\partial R}\frac{R}{PV}$$

Hicks shows that elasticity (3.11) is given by the duration formula (3.9).

B. Elasticity and Marginal Analysis

Consider now the marginal analysis of tax effects. If the negative effect of income tax just offsets the positive effect of interest tax on present value then, at the margin, the change dPV in present value equals 0:

(3.12) $$dPV = \frac{\partial PV}{\partial \tau^e} d\tau^e + \frac{\partial PV}{\partial \tau^i} d\tau^i = 0$$

Suppose, moreover, that the earnings tax rate τ^e grows at the same instantaneous rate as does the tax rate τ^i on interest:

(3.13) $$\frac{d\tau^e}{\tau^e} = \frac{d\tau^i}{\tau^i}$$

Then we can then interpret the marginal condition (3.12) on present value in terms of elasticities:

(3.14) $$\frac{\partial PV}{\partial \tau^e} \frac{\tau^e}{PV} = -\frac{\partial PV}{\partial \tau^i} \frac{\tau^i}{PV}$$

The elasticity of (after-tax) PV with respect to the earnings tax rate is:

(3.15) $$\frac{\partial PV}{\partial \tau^e} \frac{\tau^e}{PV} = -\frac{\tau^e}{1-\tau^e}$$

For a small tax rate τ_e on earnings, we can approximate this elasticity as follows:

(3.16)
$$\frac{\partial PV}{\partial \tau^e}\frac{\tau^e}{PV} \approx -\tau_e$$

On the other hand, the elasticity of PV with respect to the interest tax rate is:

(3.17)
$$\frac{\partial PV}{\partial \tau^i}\frac{\tau^i}{PV} = \frac{\partial PV}{\partial(1+r(1-\tau^i))}\frac{\partial(1+r(1-\tau^i))}{\partial \tau^i}\frac{\tau^i}{PV}$$

(3.18)
$$= -\frac{r\tau_i}{1+r(1-\tau^i)}\frac{\partial PV}{\partial(1+r(1-\tau^i))}\frac{1+r(1-\tau^i)}{PV}$$

(3.19)
$$= \frac{r\tau_i}{1+r(1-\tau^i)}D_{aftertax}$$

with $D_{aftertax}$ the variant of Macaulay-Hicks duration D based on after-tax earnings and interest:

(3.20)
$$D_{aftertax} = \frac{\sum_{t=1}^{N}\frac{t(1-\tau^e)E_t}{(1+(1-\tau^i)r)^t}}{\sum_{t=1}^{N}\frac{(1-\tau^e)E_t}{(1+(1-\tau^i)r)^t}}$$

The step (3.19) follows from Hicks' elasticity result mentioned earlier. If the tax rates are close to zero then after-tax duration is about the same as before-tax duration:

(3.21) $$D_{aftertax} \approx D$$

Applying the small-tax approximations (3.16) and (3.21), the balancing condition (3.12) is approximately:

(3.22) $$\tau^e \approx \frac{\tau^i r}{1+r} D$$

which is the Anderson and Barber breakeven condition (2.4) restated as a small-tax approximation. A minus (-) sign appears on the right-hand side of (2.4) but not (3.22). This difference in sign reflects the difference between the Anderson and Barber constant D and Macaulay-Hicks duration D.

The upshot of these analytics is that Anderson and Barbers' mathematical formulation of a "breakpoint" - for signing tax effects – seems to be a workable approximation when the relevant tax rate is sufficiently small. If so it should be possible to confirm their mathematical theory as a locally valid, in the neighborhood of a zero tax rate, though a rigorous confirmation of this sort exceeds the present scope.

IV. Concluding Remarks

The present work reconsiders the effect of tax on the present value of future growing income streams, by discussing a mathematical theory proposed by Gary Anderson and Joel Barber (2010). These authors posit that an increase in the tax rate on both earnings and interest income will lower the present value of income streams if the work period is sufficiently short, but will raise present value if the work period is sufficiently long. Moreover, they propose an exact "breakeven" point – a formula for the length of work period at which the tax effect switches from negative to positive. The present research points out that Anderson and Barbers' breakeven formula is not generally valid, but does seem to be a good approximation when the tax rate is sufficiently small.

Future research should be directed to the question of whether or not Anderson and Barbers' general conception of tax effects (on the present value of growing income streams) is valid: is there a breakeven point at which tax effects switch from negative to positive, as the work period lengthens? While a general formula for such a breakpoint is unknown, the small-tax approximation sketched in the present work provides a tentative starting point.

For added perspective on tax effects, I illustrate some possibilities in Figures 1 and 2. Each figure is a plot of without-tax and after-tax present value of earnings streams, for earnings horizons $N = 1, 2, ..., 100$. In Figure 1 I set the earnings growth rate and interest rate each equal to 0.05, and the tax rate equal to 0.5, as in the last example discussed in Section II.C and further described in Table 3. Here the tax rate is high, but the Figure supports the evidence in Table 3 of a possible "breakpoint" in tax effects at 51 years of future worklife. In Figure 2 I set $g = r = 0.03$ and $\tau = 0.2$, which might more closely match current economic conditions. Here the apparent breakpoint is at 75 years of worklife, and given that very few people work more than 75 years, tax lowers the present value of income streams for most everyone in this example. Both Figures 1 and 2 suggest that, while tax effects may switch from negative to positive as the work period lengthens, when the tax rate is not small the time to wait for the switch may be very long, in which case the effect of tax on present value may essentially be negative.

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Table 1: Present values, Tax Rate = 10 percent

N	PV without tax	PV with tax
39	39	38.67
40	40	39.76
41	41	40.86
42	42	41.96
43	43	43.06
44	44	44.17

Table 2: Present values, Tax Rate = 1 percent

N	PV without tax	PV with tax
39	39	38.980
40	40	39.989
41	41	40.999
42	42	42.009
43	43	43.019
44	44	44.030

Table 3: Present values, Tax Rate = 50 percent

N	PV without tax	PV with tax
39	39	32.749
40	40	34.060
41	41	35.403
42	42	36.780
43	43	38.190
44	44	39.630
45	45	41.110
46	46	42.630
47	47	44.180
48	48	45.770
49	49	47.400
50	50	49.060
51	51	50.770
52	52	52.520
53	53	54.320
54	54	56.150
55	55	58.035
56	56	59.962

Table 4: Breakeven Possibilities

tau	0.01	0.05	0.1	0.2	0.3	0.4	0.5
breakpoint possibility	41	42	43	44	47	49	52

Table 5: Duration

horizon	r=4%	r=5%	r=6%
1	1.000	1.000	1.000
3	2.006	2.000	1.994
6	3.528	3.500	3.472
10	5.579	5.500	5.422
15	8.179	8.000	7.823
25	13.497	13.000	12.508
50	27.485	25.500	23.533
100	58.355	50.500	42.718
infinity	infinity	infinity	106.000

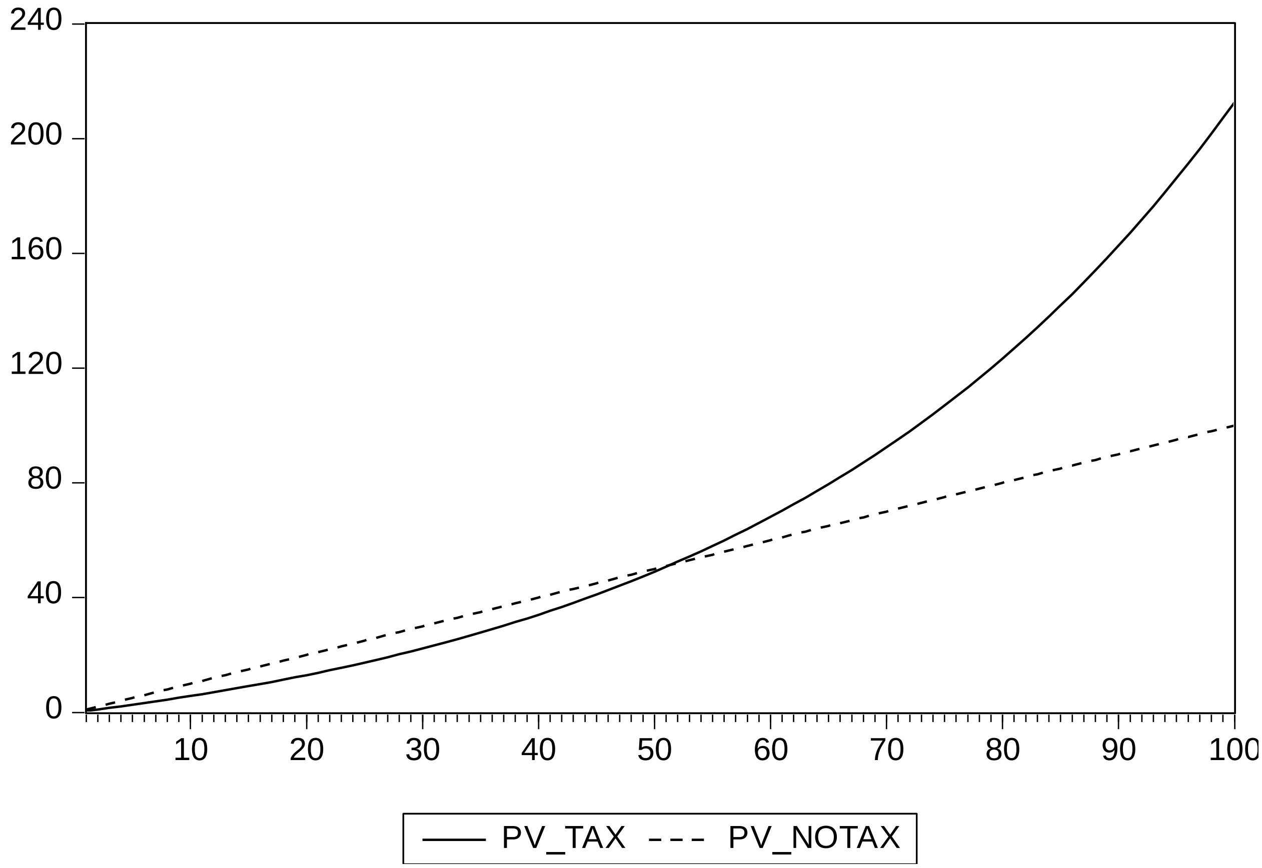

Figure 1: Present Value of Income Stream ($r = g = .05$, $\tau = .5$)

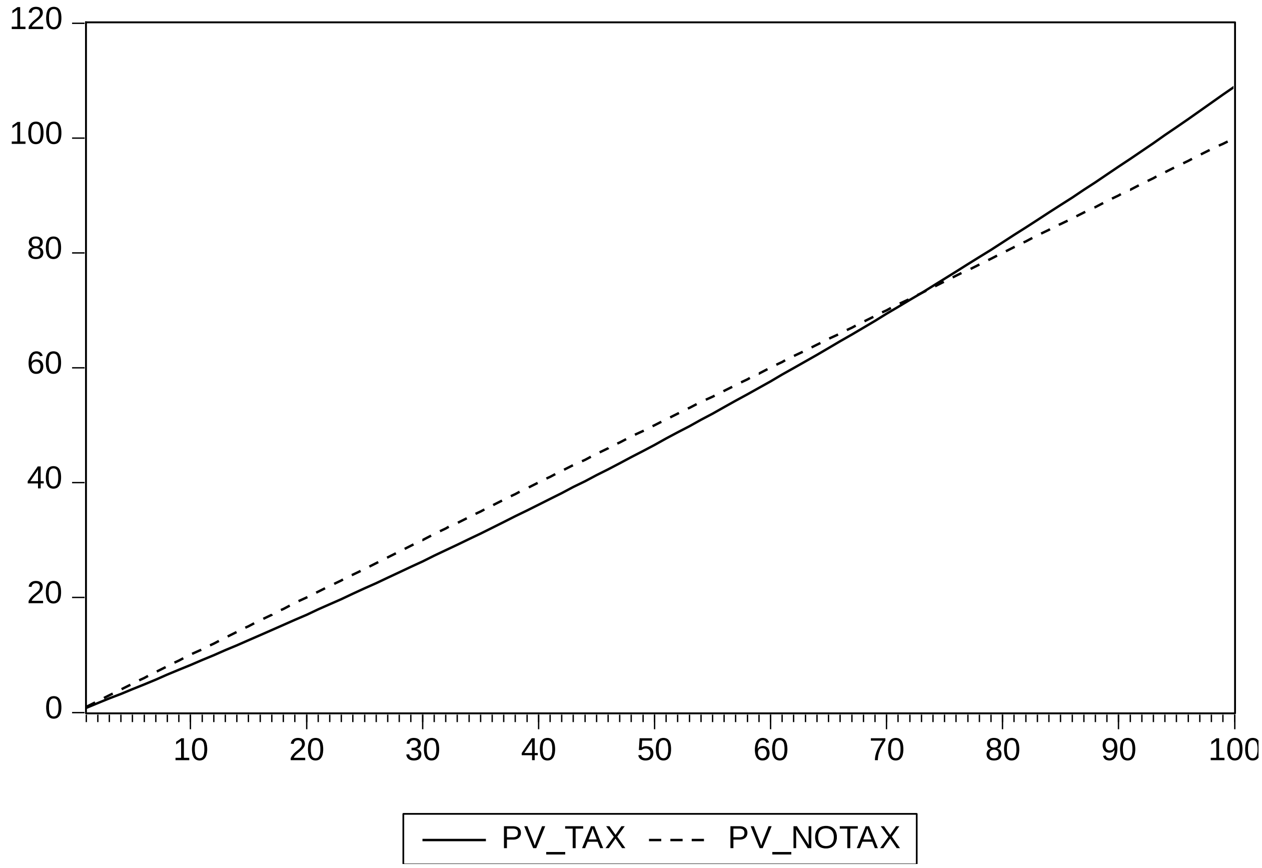

Figure 2: Present Value of Income Stream, ($r = g = .03$, $\tau = .20$)