Optimal Contracting with Unknown Risk Preference

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Abstract

In environments of uncertainty risk sharing is often an important element of economic contracts. We consider a setting where a buyer and a risk-averse supplier contract for the production of some good under cost uncertainty. At the time of contracting, both parties have incomplete information on cost of production. However, after contracting and before production, the supplier can privately discover the realization of cost. We study the supply contract that optimally balances risk sharing and information revelation when the supplier is privately informed of its risk preference. We find that all types of supplier could produce either below or above the efficient supply schedule depending on the buyer's risk preference. Moreover, "inflexible rules" rather than "discretion" arise for some range of cost realizations as a solution to the conflict between risk sharing and information revelation.

Keywords: Uncertainty; Risk Sharing; Asymmetric Information; Risk Preference

JEL Classification: D81; D82; D86

1 Introduction

Economic parties often contract in environments of uncertainty. For example, manufacturers often contract with retailers with uncertain market demand; and health insurers often contract with physicians with uncertain cost of treating patients. In these environments of uncertainty, risk sharing is often an important element of economic contracts. Optimal risk sharing has also drawn substantial attention in the agency literature. For example, Zeckhauser (1970), Spence and Zeckhauser (1971), Holmstrom (1979), Shavell (1979), and Grossman and Hart (1983) among others consider optimal risk sharing under moral hazard; Salanie (1990) studies optimal risk sharing under adverse selection; Laffont and Rochet (1998), Theilen (2003), and Dai (2008) study optimal risk sharing under both adverse selection and moral hazard. In all these studies, the equilibrium contracts closely depend on the risk preference of the contracting parties.

In reality, contracting parties seldom have perfect information on each other's risk preference. For example, the owner of a firm typically has little knowledge about its employees' degree of risk aversion; and US companies often do not have precise information on the risk preference of their Chinese suppliers. In those cases, contracting parties may have incentive to manipulate others' perception of their risk preference. For example, the suppliers may exaggerate their vulnerability to risk in order to secure more favorable contact terms. The purpose of our study is to investigate optimal risk sharing in vertical relationships where contracting parties are privately informed of their degree of risk aversion.

We consider a principal-agent relationship where a buyer contracts with a risk-averse supplier for the production of certain good. At the time of contracting, both the buyer and the supplier have incomplete information about cost of production. However, after signing the contract and before the production, the supplier can privately discover the realization of cost.

When the parties' risk preference are common information, both parties will have symmetric information at the time of contracting. Although the supplier can capture information rent after signing the contract due to its private information on the realization of cost, the buyer can fully extract the expected information rent at the time of contracting. It is well known that efficient outcomes could be achieved in this case through a fixed-price contract if the supplier were risk-neutral. However, when the supplier is risk-averse, the certainty equivalent of the supplier's ex post information rent could be different for the two parties, and the buyer may not be able to fully extract the ex post information rent at the time of contracting. In this case, the optimal supply schedule must balance risk sharing and the supplier's incentive to truthfully reveal its cost realization. We show that, when the buyer is risk-neutral, a supplier of small degree of risk aversion supplies less than the efficient level of good except for the lowest and the highest realizations of the cost. More importantly, when the supplier becomes sufficiently risk-averse, optimal contract is characterized by "rules" rather than "discretion" for some range of cost realizations. In other words, the supplier is required to produce a constant level of output for some range of cost realizations. "Rules" arise as an optimal solution to the conflict between risk sharing and the supplier's incentive to exaggerate its cost realization.

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its realization of cost but also by its degree of risk aversion. We demonstrate that the properties of the optimal contract critically depends on the buyer's risk preference.

When the buyer is risk-neutral, the optimal contract simply balances risk sharing and the incentive for the supplier to truthfully reveal both the realization of cost and its degree of risk aversion. Since the certainty equivalent of a given amount of ex post information rent is larger for a less risk-averse supplier, a less risk-averse supplier can always mimic a more risk-averse one and enjoy positive expected utility. Consequently, under the optimal contract, the supply schedule for the more risk-averse supplier is distorted further downwards to further reduce the supplier's ex post information rent. Doing so limits a less risk-averse supplier's incentive to mimic a more risk-averse one. We show that the supplier's private information on its risk preference aggregates the conflict between risk sharing and information revelation. Consequently, "rules" arise more frequently for high realizations of cost in optimal contract compared to under common information on risk preference.

When the buyer is also risk-averse, the optimal contract must simultaneously balance the buyer's surplus from different types of suppliers, risk sharing between the two parties, and the supplier's incentives for truthful information revelation. Under the optimal contract, the downward distortion in production decreases for both types of suppliers as a risk-averse buyer allocates more risk towards the suppliers. Moreover, a risk-averse buyer also reduces the production distortion for a more risk-averse supplier to smooth its surplus from different types of suppliers. Consequently, when the buyer is sufficiently risk-averse, both types of suppliers can produce above the efficient supply schedule.

Inflexible rules are commonly observed in vertical relationships. For example, in the apparel industry, retailers are often required to make firm, SKU-specific orders well in advance of the beginning of the selling season despite demonstrable advantages to in-season replenishment; in the electronics industry, flexibility for reorders is often restricted within some prespecified limits of original forecasts (Barnes-Schuster et al., 2002). Our analysis suggests that the seemingly inefficient inflexible rules can be an optimal solution to the conflict between risk sharing and information revelation in these environments of uncertainty.

Lewis and Sappington (1989a, 1989b) among others study the optimality of "inflexible rules" in agency contracts when agents face "countervailing incentives", i.e., agents have incentive to either understate or overstate their private information depending on its realization. In contrast, we show that the "inflexible rules" can arise, in the absence of countervailing incentives, as an optimal solution to the conflict between risk sharing and information revelation .

de Mezza and Webb (2000) and Jullien, Salanie and Salanie (2007) study the optimal insurance contracts under moral hazard when insurance customers are privately informed of their risk preference. Landsberger and Meilijson (1994) consider the optimal insurance contract between one risk-neutral monopolistic insurer and one risk-averse agent who is privately informed of his degree of risk aversion. Smart (2000) studies a screening game in a competitive insurance market in which insurance customers differ with respect to both accident probability and degree of risk aversion. In contrast to the above studies, we consider the optimal supply contract when suppliers differ with respect to both cost of production and degree of risk aversion.

Our study also relates to the literature on multi-period mechanism design. Riordan and Sappington (1987), Courty and Li (2000), Dai et al. (2006), and Eső and Szentes (2007) study two-period models where risk-neutral agents learn payoff-relevant private information in both periods. They analyze the optimal revelation mechanism where the contract is signed in the first period before the agent discovers his second period private information. In contrast to these articles, we study the optimal supply contract between risk-averse parties. We investigate the interaction between risk sharing and information revelation in the optimal supply contract.

The rest of the paper is organized as follows. Section 2 describes the central elements of the model. As a benchmark, Section 3 presents the optimal contract when the buyer is risk-neutral and the supplier's degree of risk aversion is common information. Section 4 examines the optimal contract when the supplier is privately informed of its degree of risk aversion. Section 5 summarizes our main findings and concludes the paper with future research directions. The proofs of all formal conclusions are in the Appendix.

2 The model

A buyer contracts with a supplier to obtain some quantity, $q \geq 0$, of a good. The buyer's valuation of q is $V(q)$, and $V(\cdot)$ is a smooth, increasing, and concave function. The buyer's surplus is $W = V(q) - T$, where T is the buyer's payment to the supplier. The supplier's total cost of producing q is $C = cq$, where c is the supplier's marginal/average cost of production. Hence, the supplier's profit is $\pi = T - cq$.

The utility of the supplier belongs to some smooth one-dimensional family of utility functions $\mathcal{F} = U_\rho(\cdot)$ that are ranked according to the Arrow-Prat measure of risk aversion: $-U''_\rho(\pi)/U'_\rho(\pi)$ is increasing with ρ for any wealth level π. Thus, ρ measures the the supplier's degree of risk aversion. The supplier's degree of risk aversion is unknown to the buyer. However, it is common knowledge that the supplier's degree of risk aversion, ρ, belongs to the two point support $\{l, h\}$ with $h > l$, $\Pr(\rho = l) = \alpha_l$ and $\Pr(\rho = h) = \alpha_h = 1 - \alpha_l$.

The supplier's marginal cost of production, c, is uncertain at the time of contracting. However, it is common knowledge that the distribution of c follows an absolutely continuous and strictly increasing cumulative distribution function $F(c)$ on $[\underline{c}, \overline{c}]$. After contracting with the buyer and before the production takes place, the supplier privately discovers the realization of c.

We assume that the distribution of c satisfies the following regularity condition: $d[c + F(c)/f(c)]/dc \geqslant 0$. The condition is commonly imposed in agency literature to ensure that the equilibrium supply schedule to be monotonically decreasing in c. As we demonstrate later in our analysis, the condition does not ensure such property in equilibrium supply schedule in our model.

The timing and contractual relation between the buyer and the supplier are as follows:

(1) the supplier privately learns its degree of risk aversion ρ; (2) the buyer offers the supplier a set of contract menus $M_\rho = \{T_\rho(c), q_\rho(c)\}$ conditional on the supplier's degree of risk aversion ρ and its realization of marginal cost c; (3) the supplier selects its preferred menu M_ρ given its private information on ρ; (4) the supplier discovers c, and selects a desired option $(T_\rho(c), q_\rho(c))$ from the selected menu M_ρ; (5) exchange takes place according to the contract terms.

3 Common Information on Risk Preference

As a benchmark, in this section we discuss the optimal contract when the buyer is risk-neutral and the supplier's degree of risk aversion is common information.

When the buyer is risk-neutral, its optimization problem is choosing $\{T_\rho(c), q_\rho(c)\}$ to maximize its expected surplus:

$$\int_{\underline{c}}^{\overline{c}} [V(q_\rho(c)) - T_\rho(c)]dF(c), \tag{1}$$

for $\rho = l,\, h$.

A contract is feasible (or implementable) provided it is *incentive compatible* and *individually rational*. Incentive compatibility requires that the contract induces each type of supplier to truthfully report its realization of marginal cost, i.e.,

$$\pi_\rho(c_i \mid c_i) \geqslant \pi_\rho(c_i \mid c_j) \text{ for } c_i \neq c_j, \tag{2}$$

where $\pi_\rho(c_i \mid c_i)$ and $\pi_\rho(c_i \mid c_j)$ denote the supplier's respective profits from choosing options $(T_\rho(c_i), q_\rho(c_i))$ and $(T_\rho(c_j), q_\rho(c_j))$ when the realization of its marginal cost is c_i. Individual rationality requires that the expected utility from the contract for each type of supplier must be nonnegative, i.e.,

$$E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))f(c)dc \geqslant 0. \tag{3}$$

Proposition 1 describes the general properties of the optimal contract when the buyer is risk-neutral and the supplier's risk preference is common information.

Proposition 1 *The optimal contract has the following properties: for $\rho = l,\ h$,*

(a) $E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(\pi_\rho(c))dF(c) = 0$*;*

(b) In no bunching region, $q_\rho(c)$ is given by

$$[V'(q_\rho(c)) - c]f(c) = F(c) - D_\rho(c), \tag{4}$$

where

$$D_\rho(c) = \frac{\int_{\underline{c}}^{c} U_\rho'(\pi_\rho(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_\rho'(\pi_\rho(x))dF(x)}; \tag{5}$$

(c) There exists σ such that complete or partial bunching occurs for some interval $[c', \overline{c}]$ when $\rho > \sigma$.

Proof. See appendix. ∎

When the supplier's degree of risk aversion is common information, both parties have symmetric information at the time of contracting. Consequently, although the supplier can capture ex post information rent from its private information on the realization of c after signing the contract, the buyer can fully extract the expected information rent at the time of contracting by reducing the level of transfer payments $T(c)$ for all realizations of c. (Note that it is the difference in $T(c)$ that provides the incentive for the supplier to truthfully reveal its marginal cost.) Consequently, the supplier receives zero expected utility under the optimal contract.

Given that the buyer can fully extract the supplier's ex post information rent at the time of contracting, the buyer does not face the traditional trade-off between rent extraction and production efficiency as in Baron and Myerson (1982). As we show below, the supplier's ex post information rent would be costless to the buyer and the efficient outcome would be achieved if the supplier were risk-neutral. However, when the supplier is risk-averse, the optimal supply schedule must balance risk sharing and the incentive for truthful information revelation. Equation (4) demonstrates the intuition.

When the supplier's realization of marginal cost is $\widetilde{c}$, raising $q_\rho(\widetilde{c})$ by δq will in expectation increase the supplier's production efficiency by $[V'(q_\rho(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})\delta q$. However, the increase in $q_\rho(\widetilde{c})$ will also raise the supplier's ex post information rent by δq when $c < \widetilde{c}$. Consequently, in expectation the increase in $q_\rho(\widetilde{c})$ raises the supplier's ex post information rent by $F(\widetilde{c})\delta q$. When the supplier is risk-averse, the buyer can only reduce $T_\rho(c)$ for all realizations of c by $\delta q D_\rho(\widetilde{c})$ in order to induce the supplier's participation. Notice that $\delta q \int_{\underline{c}}^{\widetilde{c}} U'_\rho(\pi_\rho(x))dF(x)$ is the increase in the supplier's expected utility as a result of the increase in ex post information rent, and $\int_{\underline{c}}^{\overline{c}} U'_\rho(\pi_\rho(x))dF(x)$ is the increase in the supplier's expected utility as a result of one unit of increase in $T_\rho(c)$ for all realizations of c. Therefore, $\delta q D_\rho(\widetilde{c})$ is the certainty equivalent of the increase in ex post information rent for the supplier. At the optimum, the buyer's marginal benefit of raising $q_\rho(\widetilde{c})$ must equal its marginal cost of doing so, which yields equation (4).

When the supplier is risk-neutral, i.e., $u'' = 0$, $D_\rho(\widetilde{c}) = F(\widetilde{c})$, which means the certainty equivalent of the increase in information rent is $F(\widetilde{c})$ for both the buyer and the seller. Consequently, the buyer can fully extract the supplier's expected ex post information rent by reducing the transfer payments for all realizations of c by exactly $F(\widetilde{c})\delta q$. In that case, the right-hand side of equation (4) becomes zero, and $V'(q_\rho(c)) = c$. The optimal contract

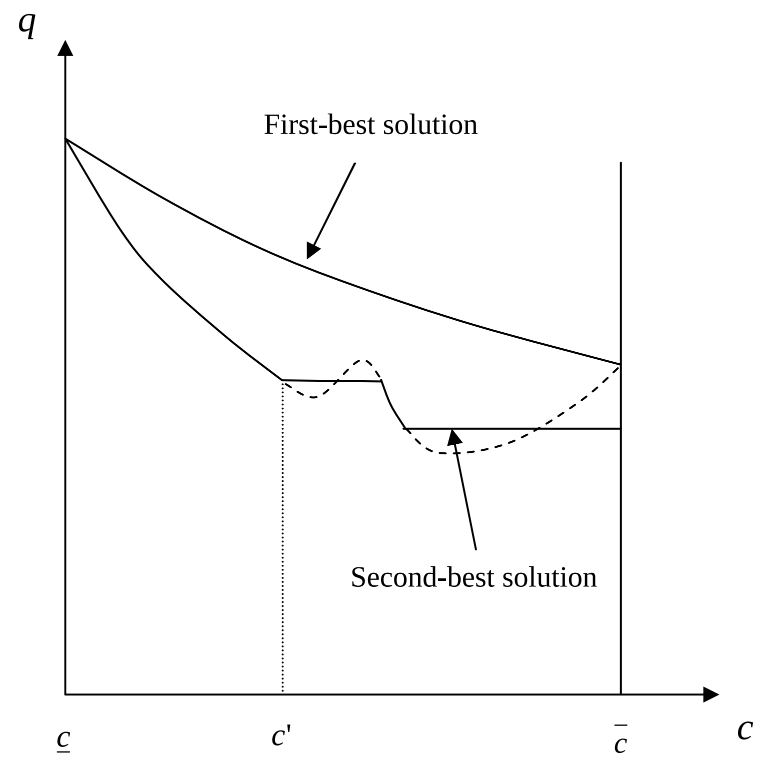

Figure 1: Optimal supply schedule with partial bunching in $[c', \overline{c}]$.

would be a fixed price contract, and the supplier would always supply the efficient level of good.

When the supplier is of small degree of risk aversion, the optimal supply schedule is strictly decreasing in c in $[\underline{c}, \overline{c}]$. Then, equation (4) suggests that $D_\rho(c) = F(c)$ and $V'(q_\rho(c)) = c$ at $\underline{c}$ and $\overline{c}$. In other words, the supplier delivers the efficient amount of good at $\underline{c}$ and $\overline{c}$. Notice that $D'_\rho(c) = U'_\rho(\pi_\rho(c))f(c)/\int_{\underline{c}}^{\overline{c}} U'_\rho(\pi_\rho(x))dF(x)$ and $\partial U'_\rho(\pi_\rho(c))/\partial c = -U''_\rho(\pi_\rho(c))q_\rho(c) > 0$. Therefore, the sign of $f(c) - D'(c)$ must change once and only once with $f(\underline{c}) - D'(\underline{c}) > 0$ and $f(\overline{c}) - D'(\overline{c}) < 0$. Since $F(c) - D_\rho(c) = 0$ at $\underline{c}$ and $\overline{c}$, it suggests that $F(c) - D_\rho(c) > 0$ on $(\underline{c}, \overline{c})$. Consequently, equation (4) suggests that the supplier delivers less than the efficient amount of good on $(\underline{c}, \overline{c})$.

When the supplier becomes sufficiently risk-averse, the monotonicity condition ($q_\rho(c)$ is non-increasing) becomes constraining. In this case, "inflexible rules" arise as an optimal solution to the conflict between risk sharing and the supplier's incentive to exaggerate its cost realization. In other word, bunching occurs and the supplier is required to produce a constant level of good in some interval $[c', \overline{c}]$ where $\underline{c} < c' < \overline{c}$. When $f(c)$ does not change rapidly (for example, c is uniformly distributed), bunching occurs for the entire interval $[c', \overline{c}]$.[1] On the other hand, when $f(c)$ does change rapidly, bunching could occur for some ranges of c in the interval $[c', \overline{c}]$. Figure 1 demonstrates the optimal supply schedule with partial bunching in some interval $[c', \overline{c}]$.

When the supplier converges to infinitely risk-averse, $D_\rho(c)$ converges to zero for $c \in [\underline{c}, \overline{c})$. Then equation (4) converges to

$$[V'(q_\rho(c)) - c]f(c) = F(c) \tag{6}$$

for $c \in [\underline{c}, \overline{c})$, which is the well known solution for a standard adverse selection problem where the supplier is privately informed of its marginal cost of production at the time of contracting. This is because the supplier will participate in the contract only if it is guaranteed nonnegative profit for all realizations of c when it is infinitely risk-averse. Consequently our model becomes equivalent to one that the supplier is perfectly informed of its marginal cost at the time of contracting.

For later use, we call the optimal supply schedule when the supplier's degree of risk aversion is common information the second-best supply schedule.

[1] With a constant absolute risk aversion (CARA) utility function and an uniform distribution of c, Salanie (1990) and Laffont and Rochet (1998) also show that complete bunching arises in some interval $[c^*, \overline{c}]$ where $\underline{c} < c^* < \overline{c}$.

4 Asymmetric Information on Risk Preference

4.1 A Risk-Neutral Buyer

When the supplier is privately informed of its degree of risk aversion, the buyer must screen the supplier not only by its realization of cost but also by its degree of risk aversion.

When the buyer is risk-neutral, the buyer's optimization problem is choosing a set of contract menus $M_\rho = \{T_\rho(c), q_\rho(c)\}$ to maximize

$$\int_{\underline{c}}^{\overline{c}} \{\alpha_l[V(q_l(c)) - T_l(c)] + \alpha_h[V(q_h(c)) - T_h(c)]\} f(c)dc, \tag{7}$$

subject to

$$E[U_\rho(M_\rho)] = \int_{\underline{c}}^{\overline{c}} U_\rho(T_\rho(c) - cq_\rho(c))f(c)dc \geqslant 0; \tag{8}$$

$$\pi_\rho(c_i \mid c_i) \geqslant \pi_\rho(c_i \mid c_j) \text{ for } c_i \neq c_j; \text{ and} \tag{9}$$

$$E[U_\rho(M_\rho)] \geqslant E[U_\rho(M_s)], \tag{10}$$

where $\rho = l$, h, $s = l$, h, and $\rho \neq s$.

The conditions (8) and (9) ensure the supplier's participation and its truthful revelation of marginal cost regardless of its degree of risk aversion; and condition (10) guarantees the supplier truthfully reveals its degree of risk aversion.

Proposition 2 describes the general properties of the optimal contract when the buyer is risk-neutral and the supplier is privately informed of its degree of risk aversion.

Proposition 2 *When the buyer is risk-neutral, the optimal contract has the following properties :*

(a) $E[U_l(M_l)] > E[U_h(M_h)] = 0$;

(b) In no bunching region, the optimal supply schedule for the less risk-averse supplier is characterized by

$$[V'(q_l(c)) - c]f(c) = F(c) - D_l(c); \text{ and} \tag{11}$$

the optimal supply schedule for the more risk-averse supplier is characterized by

$$\alpha_h[V'(q_h(c)) - c]f(c) = \alpha_h[F(c) - D_h(c)] + \alpha_l G(c), \tag{12}$$

where $D_\rho(c)$ *is defined by (5) for* $\rho = l,\ h$ *and*

$$G(c) \equiv \frac{\int_{\underline{c}}^{c} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)}. \tag{13}$$

(c) There exists σ'_ρ *such that complete or partial bunching occurs for some interval* $[c'_\rho, \overline{c}]$ *when* $\rho > \sigma'_\rho$ *for* $\rho = l,\ h$.

Proof. See Appendix. ■

Under the optimal contract, the buyer can fully extract the more risk-averse supplier's ex post information rent by adjusting the level of payments for all realizations of marginal cost as in the case of common information on risk preference. However, the utility function of a less risk-averse supplier is an increasing and convex transformation of that of a more risk-averse supplier, and the less risk-averse supplier can always enjoy positive expected utility by mimicking a more risk-averse one. Consequently, the optimal contract provides a less risk-averse supplier positive expected utility to induce its truthful revelation of its degree of risk aversion.

Under the optimal contract, the supply schedule for the less risk-averse supplier optimally balances risk sharing and the incentive for the supplier to truthfully reveal its realization of marginal cost, as in the case of common information on risk preference. Consequently, the less risk-averse supplier produces according to the second-best supply schedule.

However, the supply schedule for the more risk-averse supplier now must simultaneously tradeoff risk sharing, the supplier's incentives to truthfully reveal its realization of cost, and the less risk-averse supplier's incentive to truthfully reveal its degree of risk aversion. Equation (12) demonstrates the tradeoff.

When the more risk-averse supplier's realization of marginal cost is $\widetilde{c}$, raising $q_h(\widetilde{c})$ by δq will in expectation increase the production efficiency by $\alpha_h[V'(q_h(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})\delta q$ where α_h is probability that the supplier is more risk-averse. However, the increase in $q_h(\widetilde{c})$ will also raise the more risk-averse supplier's ex post information rent by δq when $c < \widetilde{c}$. Consequently, in expectation it increases the more risk-averse supplier's ex post information rent by $\delta q \int_{\underline{c}}^{\widetilde{c}} U_h'(\pi_h(x))dF(x)$. In addition, the increase in $q_h(\widetilde{c})$ will also raise the less risk-averse supplier's rent by $\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_h(x))dF(x)$ when it mimics the more risk-averse one.

The certainty equivalents of the above ex post information rents for both types of suppliers are $\delta q \int_{\underline{c}}^{\widetilde{c}} U_h'(\pi_h(x))dF(x)/ \int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)$ and $\delta q \int_{\underline{c}}^{\widetilde{c}} U_l'(\pi_h(x))dF(x)/ \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)$, respectively. Notice that $\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)$ and $\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)$ are the increases in expected utilities resulting from one unit increase in all possible states for both types of suppliers, respectively.

In anticipation of the supplier's ex post information rent, at the time of contracting the buyer can reduce the more risk-averse supplier's payments by the amount of $\delta q \int_{\underline{c}}^{\widetilde{c}} U_h'(\pi_h(x))dF(x)/ \int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)$ for all realizations of marginal cost. Doing so fully extracts the more risk-averse supplier's ex post information rent. The reduction in payments for the more risk-averse supplier also reduces the less risk-averse supplier's rent from exaggerating its degree of risk aversion by $\delta q \int_{\underline{c}}^{\widetilde{c}} U_h'(\pi_h(x))dF(x)/ \int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)$. Consequently, the buyer must increase the less risk-averse supplier's payment for all realizations of c by $\delta q G(\widetilde{c})$ as a result of the increase in $q_h(\widetilde{c})$. Doing so provides the less risk-averse supplier just enough incentive to truthfully reveal its degree of risk aversion. At the optimum, the supplier's marginal benefit of raising $q_h(\widetilde{c})$ must equal its marginal cost of doing so, which yields equation (12).

Notice that $\alpha G(c)$ (which is positive on $(\underline{c}, \overline{c})$ as shown in the proof of Corollary 1) is the effect of the supplier's private information on its risk preference. In order to limit a less risk-averse supplier's incentive to exaggerate its degree of risk aversion, the buyer further distorts the more risk-averse supplier's contract towards a cost-plus contract. Consequently, as we show in Corollary 1, the more risk-averse supplier produces below the second-best supply schedule.

Corollary 1 *Under the optimal contract, the more risk-averse supplier's supply schedule is below the second-best level.*

Proof. See Appendix. ■

As either type of supplier becomes sufficiently risk-averse, the monotonicity condition ($q_\rho(c)$ is non-increasing) becomes constraining and "inflexible rules" arise in the optimal contract, similar to the case of common information on risk preference. Then, the supplier is required to produce a constant level of good in some interval $[c'_\rho, \overline{c}]$ where $\underline{c} < c'_\rho < \overline{c}$ and $\rho = l,\ h$. However, as we show in Corollary 2, the supplier's private information on its risk preference aggregates the conflict between risk sharing and information revelation. Consequently, "inflexible rules" arise more frequently for high realizations of cost compared to the case of common information on risk preference.

Corollary 2 *"Inflexible rules" arise more frequently for high realizations of cost compared to common information on risk preference.*

Proof. See Appendix. ■

Suppose that one type of supplier is risk-neutral and the other type of supplier is infinitely risk-averse. Then equation (12) becomes

$$\alpha_h[V'(q_h(c)) - c]f(c) = F(c). \tag{14}$$

A direct comparison between equations (6) and (14) demonstrates the effect on the optimal contract of the supplier's private information on its risk preference. An increase in $q_h(\widetilde{c})$ by δq increases the more risk-averse supplier's production efficiency by $[V'(q(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})\delta q$ regardless whether the supplier is privately informed of its risk preference. However, with private information on risk preference, an increase in $q_h(\widetilde{c})$ by δq increases the ex post information rent for not only the more risk-averse supplier but also the less risk-averse supplier by $F(\widetilde{c})$. The certainty equivalent of the ex post information rent is zero for the more risk-averse supplier, which means that the buyer cannot extract any of the ex post rent at the time of contracting. Consequently, with private information on risk preference, the more risk-averse supplier's supply schedule is further distorted towards a cost plus contract.

4.2 A Risk-Averse Buyer

When the buyer is also risk-averse, the optimal contract must balance the buyer's surplus from different types of suppliers, in addition to the tradeoff between risk sharing and the incentives for the supplier to truthfully reveal both its realization of cost and its degree of risk aversion.

Suppose the buyer's utility function $U(\cdot)$ also belongs to some smooth one-dimensional family of utility functions that is ranked according to the Arrow-Prat measure of risk aversion ρ. The buyer's optimization problem is choosing a set of contract menus $M_\rho =$

$\{T_\rho(c), q_\rho(c)\}$ for $\rho = l$, h to maximize

$$E[U] = \int_{\underline{c}}^{\overline{c}} \{\alpha_l U(W_l(c)) + \alpha_h U(W_h(c))\} f(c)dc \tag{15}$$

subject to conditions (8), (9), and (10), where $W_l(c) = V(q_l(c)) - T_l(c)$ and $W_h(c) = V(q_h(c)) - T_h(c)$.

Proposition 2 describes the properties of the optimal contract when both the buyer and the supplier are risk-averse.

Proposition 3 *When both the buyer and the supplier are risk-averse, the optimal contract has the following properties:*

(a) $E[U_l(M_l)] > E[U_h(M_h)] = 0$;

(b) In no bunching region, the optimal supply schedule for the less risk-averse supplier is characterized by

$$\frac{\alpha_l U'(W_l(c))[V'(q_l(c)) - c]f(c)}{\alpha_l\eta_l + \alpha_h\eta_h} = \frac{\alpha_l\eta_l}{\alpha_l\eta_l + \alpha_h\eta_h}\left\{\frac{\int_{\underline{c}}^{c} U'(W_l(x))dF(x)}{\eta_l} - D_l(c)\right\}; \tag{16}$$

and the optimal optimal supply schedule for the more risk-averse supplier is characterized by

$$\frac{\alpha_h U'(W_h(c))[V'(q_h(c)) - c]f(c)}{\alpha_l\eta_l + \alpha_h\eta_h} = \frac{\alpha_h\eta_h}{\alpha_l\eta_l + \alpha_h\eta_h}\left\{\frac{\int_{\underline{c}}^{c} U'(W_h(x))dF(x)}{\eta_h} - D_h(c)\right\} \tag{17}$$

$$+\frac{\alpha_l\eta_l}{\alpha_l\eta_l + \alpha_h\eta_h}G(c), \tag{18}$$

where $\eta_l \equiv \int_{\underline{c}}^{\overline{c}} U'(W_l(x))dF(x)$ *and* $\eta_h \equiv \int_{\underline{c}}^{\overline{c}} U'(W_h(x))dF(x)$.

Proof. See Appendix. ■

Under the optimal contract, the more risk-averse supplier still receives zero expected utility, and the less risk-averse supplier still receives positive expected utility due to its private information on its degree of risk aversion. However, the optimal supply schedule is profoundly different compared to the case when the buyer is risk-neutral.

For the less risk-averse supplier, when the realization of marginal cost is $\widetilde{c}$, raising $q_l(\widetilde{c})$ by δq will increase $W_l(\widetilde{c})$ by $[V'(q_l(\widetilde{c})) - \widetilde{c}]\delta q$ which increases the buyer's certainty equivalent by $\delta q \alpha_l U'(W_l(\widetilde{c}))[V'(q_l(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})/(\alpha_l \eta_l + \alpha_h \eta_h)$. Note that $\alpha_l \eta_l + \alpha_h \eta_h$ is the increase in the buyer's expected utility resulting from one unit increase in its surplus for all possible events. On the other hand, the increase in $q_l(\widetilde{c})$ will also raise the less risk-averse supplier's ex post information rent by δq when $c < \widetilde{c}$. For the buyer, the additional ex post information rent is equivalent to a reduction of $\int_{\underline{c}}^{\widetilde{c}} U'(W_l(x))dF(x)/\eta_l$ in $W_l(c)$ for realizations of c. For the less risk-averse supplier, the certainty equivalent of the additional ex post information rent is $D_l(\widetilde{c})$ as discussed earlier. Therefore, at the time of contracting the buyer can optimally reduce the less risk-averse supplier's payments by $\delta q D_l(\widetilde{c})$ for all realizations of c. Consequently, the right-hand side of (16) is the effect of the additional ex post information rent on the buyer's certainty equivalent.

Notice that, depending on the relative sizes of $\int_{\underline{c}}^{\widetilde{c}} U'(W_l(x))dF(x)/\eta_l$ and $D_l(\widetilde{c})$, which in turn depends on the relative degree of risk aversion between the two parties, the right-hand side of (16) can be either positive or negative and the optimal supply schedule can be either below or above the efficient level.

For example, when the supplier converges to risk-neutral, $D_l(c)$ converges to $F(c)$. In the optimal contract, $W_l(c)$ must be decreasing and therefore $U'(W_l(c))$ must be increasing in c. Moreover, $\int_{\underline{c}}^{\widetilde{c}} U'(W_l(x))dF(x)/\eta_l = F(c)$ at $\underline{c}$ and $\overline{c}$. Then it can be readily show that $\int_{\underline{c}}^{c} U'(W_l(x))dF(x)/\eta_l < F(c)$ and the right-hand side of equation (16) is negative on $(\underline{c}, \overline{c})$. Consequently, the optimal supply schedule is above the efficient level on $(\underline{c}, \overline{c})$.

On the other hand, by continuity the optimal supply schedule must be below the efficient level when the buyer converges to risk-neutral, based on our analysis of a risk-neutral buyer in the previous section.

The buyer's risk aversion has a different impact on the more risk-averse supplier's supply schedule. Equation (17) demonstrates how the optimal supply schedule for the more risk-averse supplier balances risk sharing, incentives for information revelation, and the buyer's surplus from different types of suppliers.

The certainty equivalent of the additional surplus for the buyer from increasing $q_h(\widetilde{c})$ by δq is $\delta q \alpha_h U'(W_h(\widetilde{c}))[V'(q_h(\widetilde{c})) - \widetilde{c}]f(\widetilde{c})/(\alpha_l \eta_l + \alpha_h \eta_h)$. However, the increase in $q_h(\widetilde{c})$ also increases the ex post information rent for both types of suppliers. For the buyer, the additional ex post information rent is equivalent to a reduction of $\int_{\underline{c}}^{\widetilde{c}} U'(W_h(x))dF(x)/\eta_h$ in $W_h(c)$ for all realizations of c. For the more risk-averse supplier, the certainty equivalent of the additional ex post information rent is $D_h(\widetilde{c})$. Therefore, the additional ex post information rent eventually reduces the buyer's surplus from a more risk-averse supplier by $\int_{\underline{c}}^{\widetilde{c}} U'(W_h(x))dF(x)/\eta_h - D_h(\widetilde{c})$ for all realizations of c. We have shown earlier that, as a result of the increase in $q_h(\widetilde{c})$, the buyer must also increase the less risk-averse supplier's payments by $G(\widetilde{c})\delta q$ for all realizations of c to induce its truthful revelation of cost realizations. Consequently, the increase in $q_h(\widetilde{c})$ also reduces the buyer's surplus from a less risk-averse supplier by $G(\widetilde{c})\delta q$ for all realizations of c. Therefore, the right-hand side of (17) is the overall effect of the additional ex post information rent on the buyer's certainty equivalent. Notice that the ratio $\alpha_\rho \eta_\rho/(\alpha_l \eta_l + \alpha_h \eta_h)$ for $\rho = l, h$ measures how the risk-averse buyer weights the surplus from different types of suppliers.

Similar to the case of risk-neutral buyer, in order to restrict a less risk-averse supplier's incentive to exaggerate its degree of risk aversion, the buyer distorts the more risk-averse supplier's contract towards a cost plus contract compared to the contract for the less risk-averse supplier. However, the distortion is smaller compared to the case of a risk-neutral buyer as $\int_{\underline{c}}^{\overline{c}} U'(W_l(x))dF(x) < \int_{\underline{c}}^{\overline{c}} U'(W_h(x))dF(x)$, i.e., the distortion for a more risk-

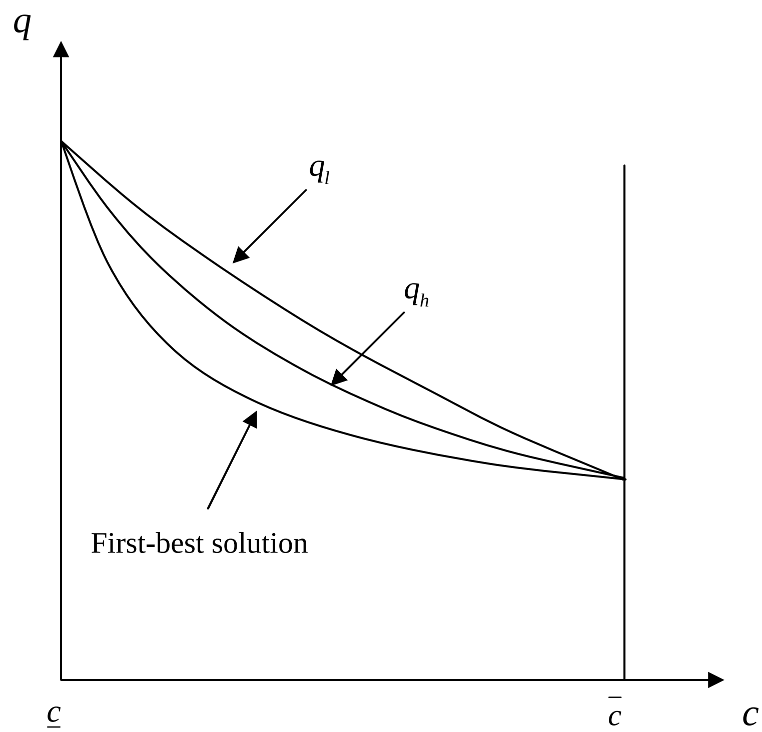

Figure 2: Optimal contract with a highly risk-averse buyer.

averse supplier becomes more costly and the information rent for a less risk-averse supplier becomes less important to a risk-averse buyer.

It can be readily shown that the optimal supply schedule for a more risk-averse supplier also can be either below or above the efficient level depending on the relative degree of risk aversion between the buyer and the supplier. Figure 2 demonstrates an optimal contract with both types of suppliers produce above the efficient supply schedule.

We summarize the above property of the optimal contract with a risk-averse buyer in Corollary 3.

Corollary 3 *Depending on the relative degree of risk aversion between the buyer and the supplier, the optimal supply schedule for both types of suppliers can be either below or above the efficient level.*

5 Conclusion

Economic parties often contract in environments of uncertainty, and risk sharing is an important element of many economic contracts. We study the supply contract between a buyer and a risk-averse supplier under cost uncertainty when the supplier is privately informed of its risk preference. We show that the optimal contract simultaneously balances risk sharing, incentives for information revelation, and the buyer's expected surplus with different types of suppliers. When the buyer is risk-neutral, all types of supplier generally produce below the efficient level of output. Moreover, the supply schedule for a more risk-averse supplier is distorted further downwards to induce a less risk-averse supplier's revelation of its risk preference. However, when the buyer is also risk-averse, both types of suppliers may produce above the efficient supply schedule.

The supplier's private information on its risk preference also aggregates the conflict between risk sharing and information revelation. Consequently, the optimal contract is often characterized by "inflexible rules" rather than "discretion" for some range of cost realizations. Therefore, our analysis shows, in the absence of transaction costs, bounded rationality, and countervailing incentives, seemingly inefficient "inflexible rules" can arise in optimal contracts in environments of uncertainty.

Our research could be extended in several directions. For example, although the supplier's information on cost of production is incomplete at the time of contracting, the supplier could be better informed of its future costs than the buyer is. Moreover, suppliers with different levels of expertise might have different forecasts of future costs at the time of contracting. The optimal contracts in these situations merit further investigation.

6 Appendix

6.1 Proof of Proposition 1

A well known characterization of feasible contracts is the following: *(a)* $T'(c) = cq'(c)$; *(b)* $q(c)$ *is non increasing;* (c) $EU \geq 0$.

Therefore, the buyer's optimization problem can be written as an optimal control problem with state variables $T(c)$ and $q(c)$ and control variable $q'(c) = z$:

$$Max \int_{\underline{c}}^{\overline{c}} [V(q(c)) - T(c)]dF(c), \tag{A1}$$

subject to

$$q'(c) = z \tag{A2}$$

$$T'(c) = cz \tag{A3}$$

$$q'(c) \leq 0; \text{ and} \tag{A4}$$

$$\int_{\underline{c}}^{\overline{c}} U(T(c) - cq(c))dF(c) \geqslant 0. \tag{A5}$$

The Hamiltonian is

$$H = [V(q) - T]f(c) + \mu cz + \lambda z + \theta U(\pi)f(c). \tag{A6}$$

The necessary conditions are given by

$$\frac{\partial H}{\partial z} = \mu c + \lambda \geqslant 0, \; z \leqslant 0, \text{ and } (\mu c + \lambda)\, z = 0; \tag{A7}$$

$$\lambda' = -\frac{\partial H}{\partial q} = -[V'(q) - \theta U'(\pi)c]f(c); \tag{A8}$$

$$\mu' = -\frac{\partial H}{\partial T} = -[-1 + \theta U'(\pi)]f(c); \text{ and} \tag{A9}$$

$$\lambda(\underline{c}) = \lambda(\overline{c}) = \mu(\underline{c}) = \mu(\overline{c}) = 0. \tag{A10}$$

From (A9) and (A10),

$$\mu(\overline{c}) - \mu(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [1 - \theta U'(\pi)]dc = 0. \tag{A11}$$

Therefore,

$$\theta = \frac{1}{\int_{\underline{c}}^{\overline{c}} U'(\pi(c))dF(c)}. \tag{A12}$$

Define $h(c) = \mu c + \lambda$. From condition (A7), on any interval where q is strictly decreasing, $h(c)$ and $\lambda(c)$ must be zero. So $h'(c) = \mu + \mu' c + \lambda' = 0$, which leads to

$$\mu = -\mu' c - \lambda'. \tag{A13}$$

Since $\lambda'(c) = 0$, substituting (A9) into the above equation for μ' and λ', we have

$$\mu = \int_{\underline{c}}^{c} [1 - \theta U'(\pi)]f(x)dx = [V'(q) - c]f(x). \tag{A14}$$

Substituting (A12) into the above equation for θ, we have

$$[V'(q) - c]f(c) = F(c) - \frac{\int_{\underline{c}}^{c} U'(\pi(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)}. \tag{A15}$$

Since $1 - U'(\pi(c))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ is strictly decreasing in c, its sign changes once and only once with $1 - U'(\pi(c))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x) > 0$ (< 0)at $c = \underline{c}$ ($c = \overline{c}$). Moreover, $F(c) - \int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ equals zero at $\underline{c}$ and $\overline{c}$. Therefore, $F(c) - \int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ must be positive on $(\underline{c}, \overline{c})$.

From (A15), we have

$$\frac{dV'(q)}{dc} = 2 - \frac{U'(\pi(c))}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)} - \frac{f'(c)}{f^2(c)}[F(c) - \frac{\int_{\underline{c}}^{c} U'(\pi(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)}]. \tag{A16}$$

Since $\int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ equals zero at $\underline{c}$ and $U'(\pi(c))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x) <$ 1 in the neighborhood of $\underline{c}$, (A16) suggests $dV'(q)dc > 0$ and the monotonicity condition is not constraining in the neighborhood of $\underline{c}$.

When the supplier converges to risk-neutral, $U'(\pi(c))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ converges to 1, $\int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ converges to $F(c)$, and $dV'(q)dc$ converges to 1. In that case, (A16) suggests that $dV'(q)dc > 0$ and q is strictly decreasing on $[\underline{c}, \overline{c}]$.

For any monotonically decreasing schedule of $q(c)$ satisfying (A15), a lower bound for $U'(\pi(\overline{c}))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ and $F(c) - \int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$, respectively, can be obtained by setting $q(c) = q(\overline{c})$ for all $c \in [\underline{c}, \overline{c}]$. It can be readily shown that the lower bound for $U'(\pi(\overline{c}))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ is great than 2 and therefore $U'(\pi(\overline{c}))/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ > 2 when the supplier is sufficiently risk averse. In that case, $dV'(q)dc < 0$ and bunching occurs for some entire interval of $[c', \overline{c}]$ if $f'(c) = 0$. However, if $|f'(c)/f^2(c)|$ is sufficiently large for some ranges of c, $dV'(q)dc < 0$ can occur at the above lower bound of $F(c) - \int_{\underline{c}}^{c} U'(\pi(x))dF(x)/\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)$ for some middle range of c., i.e., bunching can occur for some middle range of c.

For any bunching range $[c_1, c_2]$ in $[c', \overline{c}]$, $\lambda(c_1) = \lambda(c_2) = 0$ for continuity. Therefore, from (A8) we have

$$\int_{c_1}^{c_2} [V'(q) - \frac{U'(\pi(c))c}{\int_{\underline{c}}^{\overline{c}} U'(\pi(x))dF(x)}]dc = 0.$$

Moreover, $q(c_1)$ and $q(c_2)$ are determined by (A15).

6.2 Proof of Proposition 2

The Hamiltonian is

$$\begin{aligned} H &= \alpha[V(q_l(c)) - T_l(c)]f(c) + (1-\alpha)[V(q_h(c)) - T_h(c)]f(c) && \text{(A17)} \\ &+\mu_l c z_l + \mu_h c z_h + \lambda_l z_l + \lambda_h z_h + \theta U_h(\pi_h)f(c) + \beta[U_l(\pi_l) - U_h(\pi_h)]f(c), && \text{(A18)} \end{aligned}$$

where μ_l, μ_h, λ_l, λ_h, θ, and β are the Lagrange multipliers.

The necessary conditions are given by

$$\frac{\partial H}{\partial z} = \mu_l c + \lambda_l \geqslant 0,\ \ z_l \leqslant 0,\ \text{ and } (\mu_l c + \lambda_l)\ z_l = 0; \tag{A19}$$

$$\frac{\partial H}{\partial z} = \mu_h c + \lambda_h \geqslant 0,\ \ z \leqslant 0,\ \text{ and } (\mu_h c + \lambda_h)\ z_l = 0; \tag{A20}$$

$$\lambda_l' = -\frac{\partial H}{\partial q_l} = -[\alpha V'(q_l) - \beta U_l'(\pi_l)c]f(c); \tag{A21}$$

$$\lambda_h' = -\frac{\partial H}{\partial q_h} = -[(1-\alpha)V'(q_h) - \theta U_h'(\pi_h)c + \beta U_l'(\pi_h)c]f(c); \tag{A22}$$

$$\mu_l' = -\frac{\partial H}{\partial T_l} = -[-\alpha + \beta U_l'(\pi_l)]f(c); \tag{A23}$$

$$\mu_h' = -\frac{\partial H}{\partial T_h} = -[-(1-\alpha) + \theta U_h'(\pi_h) - \beta U_l'(\pi_h)]f(c);\ \text{and} \tag{A24}$$

$$\lambda_\rho(\underline{c}) = \lambda_\rho(\overline{c}) = \mu_\rho(\underline{c}) = \mu_\rho(\overline{c}) = 0,\ \text{where } \rho = l, h. \tag{A25}$$

From the transversality condition (A25) and equation (A23),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha - \beta U_l'(\pi_l)] dF(c) = 0, \tag{A26}$$

which provides

$$\beta = \frac{\alpha}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(x)} > 0. \tag{A27}$$

From equation (A24) and the transversality condition (A25),

$$\mu_h(\overline{c}) - \mu_h(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [(1-\alpha) - \theta U_h'(\pi_h) + \beta U_l'(\pi_h)] dc = 0, \tag{A28}$$

which provides

$$\theta = \frac{1-\alpha}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x)) dF(x)} - \frac{\alpha \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x)) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x)) dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x)) dF(x)}. \tag{A29}$$

When q_ρ is strictly decreasing in c, we have

$$h_\rho'(c) = \mu_\rho + \mu_\rho' c + \lambda_\rho' = 0 \text{ or } \mu_\rho = -\mu_\rho' c - \lambda_\rho'. \tag{A30}$$

Then substituting μ_ρ' and λ_ρ' into (A30), we have

$$\mu_l = \int_{\underline{c}}^{c} [\alpha - \frac{\alpha U_l'(\pi_l)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(x)}] dz = \alpha[V'(q_l(c)) - c] f(c) \tag{A31}$$

and

$$\mu_h = \int_{\underline{c}}^{c} [(1-\alpha) - \theta e^{-\rho_h \pi_h} \rho_h + \beta e^{-\rho_l \pi_h} \rho_l] dx \tag{A32}$$

$$= (1-\alpha)\left[F(c) - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)}\right] + \alpha G(c)$$

$$= (1-\alpha)[V'(q_h(c)) - c],$$

where

$$G(c) \equiv \frac{\int_{\underline{c}}^{c} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)}. \tag{A33}$$

6.3 Proof of Corollary 1

Since

$$G(c) \equiv \frac{\int_{\underline{c}}^{c} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)}, \tag{A34}$$

$$G'(c) = \frac{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x))dF(x)} R(c) f(c), \tag{A35}$$

where

$$R(c) = \left[\frac{U_l'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)} - \frac{U_h'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)}\right] \tag{A36}$$

Since $U_h(\cdot)$ must be a strictly concave transformation of $U_l(\cdot)$, there exists a strictly concave function $Y(\cdot)$ such that $U_h(\cdot) \equiv Y(U_l(\cdot))$. Therefore,

$$R(c) = \left[\frac{1}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x))dF(x)} - \frac{Y'(U_l(\pi_h(c)))}{\int_{\underline{c}}^{\overline{c}} Y'(U_l(\pi_h(x)))U_l'(\pi_h(x))dF(x)} \right] U_l'(\pi_h(c)). \tag{A37}$$

Notice that $\pi_h(c)$ is strictly decreasing in c, consequently $Y'(U_l(\pi_h(c)))$ is strictly increasing in c. Therefore, there exists some c_0 such that $R(c) > 0$ in $[\underline{c}, c_0)$ and $R(c) < 0$ in $(c_0, \overline{c}]$.

Therefore, $G'(c) > 0$ in $[\underline{c}, c_0)$ and $G'(c) < 0$ in $(c_0, \overline{c}]$. Moreover, $G(\underline{c}) = G(\overline{c}) = 0$. Consequently, $G(c) > 0$ on $(\underline{c}, \overline{c})$.

6.4 Proof of Corollary 2

From (A32), we have

$$\begin{aligned} \frac{dV'(q_h(c))}{dc} &= (1-\alpha)\left\{ 2 - \frac{U_h'(\pi_h(c))}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)} - \frac{f'(c)}{f^2(c)}[F(c) - \frac{\int_{\underline{c}}^{c} U_h'(\pi_h(x))dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x))dF(x)}] \right\} \\ &\quad +\alpha G'(c). \end{aligned} \tag{A38}$$

As we show in the proof of Corollary 1, there exists some c_0 such that $G'(c) > 0$ in $[\underline{c}, c_0)$ and $G'(c) < 0$ in $(c_0, \overline{c}]$. Therefore, the monotonicity condition is not constraining in the neighborhood of $\underline{c}$. However, for any given $F(c)$, bunching is more likely to occur in the interval $(c_0, \overline{c}]$ compared to the case of common information on risk preference.

6.5 Proof of Proposition 3

The Hamiltonian is

$$\begin{aligned} H &= \{\alpha_l U_l(W_l(c)) + \alpha_h U_h(W_h(c))\} f(c) + \mu_l c z_l + \mu_h c z_h \\ &\quad + \lambda_l z_l + \lambda_h z_h + \theta U_h(\pi_h) f(c) + \beta[U_l(\pi_l) - U_h(\pi_h)] f(c), \end{aligned} \tag{A39}$$

where μ_l, μ_h, λ_l, λ_h, θ, and β are the Lagrange multipliers.

The necessary conditions are given by

$$\frac{\partial H}{\partial z_l} = \mu_l c + \lambda_l \geqslant 0,\ \ z_l \leqslant 0,\ \text{ and } (\mu_l c + \lambda_l)\ z_l = 0; \tag{A40}$$

$$\frac{\partial H}{\partial z_h} = \mu_h c + \lambda_h \geqslant 0,\ \ z \leqslant 0,\ \text{ and } (\mu_h c + \lambda_h)\ z_l = 0; \tag{A41}$$

$$\lambda_l' = -\frac{\partial H}{\partial q_l} = -[\alpha_l U'(W_l) V'(q_l) - \beta U_l'(\pi_l) c] f(c); \tag{A42}$$

$$\lambda_h' = -\frac{\partial H}{\partial q_h} = -[\alpha_h U'(W_h) V'(q_h) - \theta U_h'(\pi_h) c + \beta U_l'(\pi_h) c] f(c); \tag{A43}$$

$$\mu_l' = -\frac{\partial H}{\partial T_l} = -[-\alpha_l U'(W_l) + \beta U_l'(\pi_l)] f(c); \tag{A44}$$

$$\mu_h' = -\frac{\partial H}{\partial T_h} = -[-\alpha_h U'(W_h) + \theta U_h'(\pi_h) - \beta U_l'(\pi_h)] f(c);\ \text{ and} \tag{A45}$$

$$\lambda_\rho(\underline{c}) = \lambda_\rho(\overline{c}) = \mu_\rho(\underline{c}) = \mu_\rho(\overline{c}) = 0, \text{ where } \rho = l, h. \tag{A46}$$

From the transversality condition (A46) and equation (A44),

$$\mu_l(\overline{c}) - \mu_l(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha_l U'(W_l) - \beta U_l'(\pi_l)] dx = 0, \tag{A47}$$

which provides

$$\beta = \frac{\alpha_l \int_{\underline{c}}^{\overline{c}} U'(W_l) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(x)}. \tag{A48}$$

From the transversality condition (A46) and equation (A45),

$$\mu_h(\overline{c}) - \mu_h(\underline{c}) = \int_{\underline{c}}^{\overline{c}} [\alpha_h U'(W_h) - \theta U_h'(\pi_h) + \beta U_l'(\pi_h)] dF(x) = 0, \tag{A49}$$

which provides

$$\theta = \frac{\alpha_h \int_{\underline{c}}^{\overline{c}} U'(W_h) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x)) dF(x)} - \frac{\alpha_l \int_{\underline{c}}^{\overline{c}} U'(W_l) dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_h(x)) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_h'(\pi_h(x)) dF(x) \int_{\underline{c}}^{\overline{c}} U_l'(\pi_l(x)) dF(x)}. \tag{A50}$$

When q_ρ is strictly decreasing in c, we have

$$h_\rho'(c) = \mu_\rho + \mu_\rho' c + \lambda_\rho' = 0 \text{ or } \mu_\rho = -\mu_\rho' c - \lambda_\rho'. \tag{A51}$$

Then substituting μ_ρ' and λ_ρ' into (A51), we have

$$\begin{aligned} \mu_l &= \int_{\underline{c}}^{c} [\alpha_l U'(W_l) - \beta U_l'(\pi_l)] dx \qquad \text{(A52)} \\ &= \int_{\underline{c}}^{c} [\alpha_l - \frac{\alpha_l U_l'(\pi_l) \int_{\underline{c}}^{\overline{c}} U'(W_l) dF(x)}{\int_{\underline{c}}^{\overline{c}} U_l'(\pi_l) dF(x)}] dx \\ &= \alpha_l U'(W_l)[V'(q_l(c)) - c], \end{aligned}$$

and

$$\mu_h = \int_{\underline{c}}^{c} [\alpha_h U'(W_h) - \theta U_h'(\pi_h) + \beta U_l'(\pi_h)] dF(x) \tag{A53}$$

$$= \alpha_h \int_{\underline{c}}^{\overline{c}} U'(W_h) dF(x) \left\{ \frac{\int_{\underline{c}}^{c} U'(W_h(x)) dF(x)}{\int_{\underline{c}}^{\overline{c}} U'(W_h) dF(x)} - D_h(c) \right\} + \alpha_l G(c) \int_{\underline{c}}^{\overline{c}} U'(W_l) dF(x)$$

$$= \alpha_h U'(W_h)[V'(q_h(c)) - c].$$

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