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An Oligopolistic Heckscher-Ohlin Model of Foreign Direct Investment

By

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Abstract

We develop a two-country, two-good, and two-factor model of international trade in which one of the sectors is perfectly competitive and the other one is oligopolistic. The oligopoly sector consists of a given number of identical firms for each country, but they are free to locate in any of the two countries. The allocation of the firms between the two countries is endogenously determined, and changes in factor prices play a crucial role in establishing this equilibrium. Under this framework we examine some of the traditional trade-theoretic issues and also carry out two comparative static exercises.

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1 Introduction

The Heckscher-Ohlin (HO) model is one of the most celebrated models of international trade theory.[1] Even in various policy discussions on international trade related issues, some of the results of the HO models are often used to support or refute a particular point of view. In recent years, this model has been extended to allow for oligopolistic interdependence (domestic and international) in production. Many researchers have examined if the traditional HO results carry over in the presence of oligopoly firms. Markusen (1981) considers international duopoly as the basis of the interdependence and finds that most of the traditional results like factor price equalization and gains from trade may not hold in the extended HO model. In sharp contrast, Lahiri and Ono (1995) allow free entry and exit of firms in the oligopoly sector in each of the two countries and show that most of the traditional HO results go through in the new framework.[2]

There is an independent theoretical literature on foreign direct investment (FDI) which also uses imperfect competition — Cournot oligopoly or monopolistic competition — as the framework of analysis.[3] However, the framework of analysis for these types of models is mostly partial equilibrium in nature, and factor prices are typically exogenous in these models. Furthermore, the issues analyzed in models of FDI are also different from the issues discussed in trade-theoretic models. There is also a small literature which examines FDI in trade-theoretic frameworks.[4] These models mostly consider monopolistic competition and FDI in the form of new plants by multi-plant multinational corporations. In contrast, we consider FDI in the form of complete relocation of firms. Furthermore, the present paper

[1]See Stolper and Samuelson (1941) and Samuelson (1948) for early formalizations of the HO model. For a historical account of the development of this model, see Jones (2004).

[2]See also, among others, Shimomura (1998) and Fujiwara and Shimomura (2005) for contributions to this field.

[3]Lahiri and Ono, 1998; Glass and Saggi, 1999. There is also a substantial literature on foreign investment in a competitive trade-theoretic models. For this, see, for example, Jones *et al.*, 1983; Ruffin, 1984.

[4]See, for example, Helpman, 1984a; Ethier, 1986; Ethier and Horn, 1990; Horstmann and Markusen, 1992.

also analyzes somewhat different issues from the literature.

This paper develops an oligopolistic HO model of FDI and defines a FDI equilibrium where factor prices are endogenous and in fact play a crucial role in establishing the equilibrium. To be more precise, it is a two-country, two-good, two-factor HO model in which one of the two sectors is oligopolistic. The number of the oligopoly firms that belong to each country is given but they can freely move between the two countries looking for higher profits, and in the equilibrium profits in the two countries are equated. This equilibrium is achieved because of the flexibility of the factor prices, i.e., the factor prices work as the equilibrating force.

Under the above framework we reexamine some of the traditional results of the standard HO model. First we examine if the factor price equalization result still holds. Second, we consider if relative factor endowments in the two countries continue to be the sole determinants of the pattern of trade in goods. Third, we analyze the issue of gains from trade for the two countries. Finally, we examine what determines the pattern of FDI in this framework. Furthemore, we apply the model to analyzing the effects of changes in the production side (technological progress) and of a shift in the preference for the oligopoly good on the incidence of FDI.

The layout of the paper is as follows. The following section sets up the model. Section 3 then is divided into several subsections to examine the trade-theoretic issues mentioned above. In section 4 we carry out the two comparative static exercises. Finally, some concluding remarks are made in section 5.

2 An HO Model of FDI

In our model there are two countries called the home and the foreign country. Each country has two sectors: one is competitive and the other is Cournot oligopolistic. Both the competitive and the oligopoly good are homogeneous and their international markets are fully integrated. In the oligopoly sector there are N firms in all: $\bar{n}$ firms are owned by the home country and $\bar{n}^*$ $(= N - \bar{n})$ by the foreign country. Of the N firms, n are located in the home country and $N - n$ in the foreign country. The international allocation of them is endogenously determined. Both commodities are produced from two factors, labor L and capital K, using constant-returns-to-scale technologies.

The production technology in the competitive sector and that of the oligopoly sector are respectively the same across the two countries:

$$A = L_a \theta f(k_a), \quad A^* = L_a^* \theta f(k_a^*), \tag{1}$$

$$x = L_o g(k_o), \quad x^* = L_o^* g(k_o^*), \tag{2}$$

$$\text{where} \qquad k_i = K_i/L_i, \quad k_i^* = K_i^*/L_i^* \ \text{ for } i = a,\ o,$$

and θ is a Hicks-neutral technological parameter in the competitive sectors of the two countries. Varibles with superscript $*$ denote those for the foreign country. Cost-minimizing behavior in the two sectors gives

$$\begin{aligned} \omega(= w/r) &= f(k_a)/f'(k_a) - k_a = g(k_o)/g'(k_o) - k_o, \\ \omega^*(= w^*/r^*) &= f(k_a^*)/f'(k_a^*) - k_a^* = g(k_o^*)/g'(k_o^*) - k_o^*, \end{aligned} \tag{3}$$

where w and r are respectively the wage rate and the capital rent. They give us the following functions:

$$k_a = k_a(\omega), \quad k_o = k_o(\omega), \quad k_a^* = k_a(\omega^*), \quad k_o^* = k_o(\omega^*), \quad \text{with} \ \ k_a' > 0, \ k_o' > 0. \tag{4}$$

Without loss of any generality, we assume that the marginal cost (price) equals unity in the competitive sector. Therefore, from (1),

$$\begin{aligned} \theta f(k_a(\omega)) &= r(\omega + k_a(\omega)), \quad \theta f(k_a(\omega^*)) = r^*(\omega^* + k_a(\omega^*)), \qquad (5) \\ \text{which yield} \qquad w &= w(r), \quad w^* = w(r^*), \\ \text{with} \qquad w'(r) &= -k_a(\omega) < 0, \quad w'(r^*) = -k_a(\omega^*) < 0. \end{aligned}$$

Since $w'(r) = -k_a(\omega)$, we obtain

$$\frac{d\omega}{dr} = \frac{1}{r} \cdot \frac{dw}{dr} - \frac{w}{r^2} = -\frac{k_a + \omega}{r}. \tag{6}$$

Under the technology given by (2) each oligopoly firm has the following marginal cost c (c^*):

$$\begin{aligned} c &= r(\omega + k_o(\omega))/g(k_o(\omega)) = c(w, r), \\ c^* &= r^*(\omega^* + k_o(\omega^*))/g(k_o(\omega^*)) = c^*(w^*, r^*) = c(w^*, r^*), \end{aligned} \tag{7}$$

and maximizes profit π (π^*):[5]

$$\pi = (p - c)x, \quad \pi^* = (p - c^*)x^*, \tag{8}$$

where p is the price of the oligopoly good.

Under free trade the inverse demand function for the oligopoly good is

$$p = p(D + D^*, Y, Y^*), \tag{9}$$

where Y and Y^* are national income, and D and D^* are demand for the oligopoly good, in the two countries. Given the profit functions in (8) and demand function (9), the first-order profit maximizing conditions for the Cournot-oligopoly firms are

$$p + p_X x = c, \quad p + p_X x^* = c^*, \tag{10}$$

where $p_X = \partial p(X, Y, Y^*)/\partial X < 0$.

Since the N oligopoly firms freely move between the two countries so as to realize higher profits, equilibrium profits in the two countries — π and π^* — are the same:

$$\pi = \pi^*. \tag{11}$$

We shall call this the FDI equilibrium. Substituting (10) into (8) and using (11) yields

$$\pi = -p_X x^2 = -p_X x^{*2} = \pi^*,$$

[5] We ignore fixed costs here for avoiding added complications. However, following the analysis in Lahiri and Ono (1995), these costs can easily be included without affecting any of our results.

which implies that $x = x^*$. Furthermore, by applying this property to (10) we find that $c = c^*$. Formally,

Lemma 1 $\quad x = x^*, \quad c = c^*,$

i.e., the marginal costs and output levels of all oligopoly firms are the same in the FDI equilibrium.

Since firms can move freely between the two countries and the oligopoly-good markets in the two countries are fully integrated, they move to the other country if marginal costs are higher in their original location. It in turn affects the factor prices in the two countries and brings the marginal cost level closer to each other's. It is thus the free international movement of the oligopoly firms that works as the equilibrating force in our model.

National income in each country is

$$Y = w\bar{L} + r\bar{K} + \bar{n}\pi, \quad Y^* = w^*\bar{L}^* + r^*\bar{K}^* + \bar{n}^*\pi, \tag{12}$$

where $(\bar{L}, \bar{K})$ and $(\bar{L}^*, \bar{K}^*)$ are the factor endowments in the two countries. The equilibrium conditions of the factor markets are

$$\begin{aligned} L_a + nL_o = \bar{L}, \quad L_a^* + (N-n)L_o^* = \bar{L}^*, \\ L_a k_a(\omega) + nL_o k_o(\omega) = \bar{K}, \quad L_a^* k_a(\omega^*) + (N-n)L_o^* k_o(\omega^*) = \bar{K}^*. \end{aligned} \tag{13}$$

Following the Heckscher-Ohlin tradition, we assume away the possibility of factor-intensity reversal and require the factor endowments of both countries to be located in the cone of diversification. Formally,

Assumption 1 *Either*

$$k_a(\omega) > \bar{k} \ (\equiv \bar{K}/\bar{L}) > k_o(\omega) \quad \textit{and} \quad k_a(\omega^*) > \bar{k}^* \ (\equiv \bar{K}^*/\bar{L}^*) > k_o(\omega^*),$$

$$\textit{or} \qquad k_a(\omega) < \bar{K}/\bar{L} < k_o(\omega) \quad \textit{and} \quad k_a(\omega^*) < \bar{K}^*/\bar{L}^* < k_o(\omega^*).$$

3 International Trade Theory and FDI

This section examines some of the traditional trade issues like factor price equalization, the pattern of trade, and gains from trade in the present setup. It also introduces a new issue absent in the traditional analysis, viz., the pattern of foreign direct investment.

3.1 Factor Price Equalization

The validity of factor price equalization is examined in the present context. From (7) and lemma 1 we find

$$c(w(r), r) = c(w(r^*), r^*).$$

Furthermore, totally differentiating the first equation of (7) and using (3) and (6) produces

$$\frac{dc}{dr} = -\frac{k_a - k_o}{g}, \tag{14}$$

which is either monotonically increasing or monotonically decreasing under assumption 1. Thus, the values of r and r^* satisfying $c(w(r), r) = c(w(r^*), r^*)$ must be the same since the functional forms of $c(\cdot)$ and $w(\cdot)$ are the same in the two countries. Since there is a one-to-one relationship between w and r, the values of w and w^* must also be the same. Then from (4) it follows that the capital-labor ratios in the two countries are the same for each sector. Therefore, from (2) and lemma 1, the amount of labor employed by each oligopoly firm is also the same, i.e., $L_o = L_o^*$. Formally,

Proposition 1 *Under assumption 1, free trade leads to factor price equalization in an Heckscher-Ohlin world where one of the two sectors is a Cournot oligopoly sector and the oligopoly firms can move freely between the two countries. All the oligopoly firms hire the same amount of labor and produce the same amount of output.*

In the standard HO model free mobility of goods between the two countries acts as a perfect substitute for factor mobility, resulting in the equalization of factor prices between the two countries. In contrast, Markusen (1981) shows that factor price equalization does not hold in a model of international duopoly. That is, in the presence of Cournot duopoly free mobility of goods is an imperfect substitute for factor mobility between countries. Lahiri and Ono (1995) find that it is not Cournot oligopoly that makes goods mobility an imperfect substitute for factor mobility, but that it is the fixity of the number of firms that does it. By introducing free entry and exist of oligopoly firms in the two countries they restore factor price equalization in the oligopolistic HO model. Proposition 1 states that even when the total number of firms in the oligopoly sector is fixed, their free mobility between the two countries allows factor price equalization to hold.

3.2 Pattern of Trade

The traditional theory on the pattern of trade now needs to be extended in two directions. First, apart from trade in goods here we have to consider trade in firms (FDI) as well. Second, apart from the endowments of capital and labor, here we have 'endowments' of firms as well. Let us first consider the pattern of FDI.

From (13) and proposition 1 we obtain

$$\frac{n}{N} = \frac{1}{1 + \frac{\bar{L}^*}{\bar{L}} \cdot \frac{\bar{k}^* - k_a}{\bar{k} - k_a}}. \tag{15}$$

This equation directly gives the following proposition:

Proposition 2 *Under assumption 1 we obtain the following result on the pattern of FDI.*

1. *If $\bar{n}^*/\bar{n} = \bar{L}^*/\bar{L}$, then $\bar{n} \gtreqless n$ according as $\bar{k}^* \gtreqless \bar{k}$.*

2. *If $\bar{n}^*/\bar{n} = \alpha(\bar{L}^*/\bar{L})$ and $\bar{k} = \bar{k}^*$, then $n \gtreqless \bar{n}$ according as $\alpha \gtreqless 1$.*

The first result states that when the relative endowments of firms and labor are the same in the two countries, a country will be a net recipient of FDI if and only if it is relatively more capital abundant. The second result establishes that when the capital-labor ratio is the same between the two countries, a country will be a net recipient of FDI if and only if the ratio of of firm endowment to labor endowment (or capital endowment) is lower in that country that in the other country. Intuitively, the above results can be explained as follows. Since each firm in the two countries employs the same amount of labor, as stated in proposition 1, an international difference in the labor employment of the oligopoly sector is caused by a difference in the number of oligopoly firms located in the two countries, which represents the pattern of FDI. Furthermore, it is shown from (13) that the ratio of labor employed in the oligopoly sector to labor endowment is higher in a country which has a higher capital-labor endowment ratio (this property is also true in the standard HO model). These two properties together give us the results of proposition 2.

Turning now to the question on the pattern of goods trade, since $\omega = \omega^*$, $L_o = L_o^*$ and $x = x^*$ as stated in proposition 1, from (13) we derive

$$\frac{L_a \theta f(k_a(\omega))}{nx} - \frac{L_a^* \theta f(k_a(\omega^*))}{n^* x^*} = \theta L_o (k_a - k_o) \cdot \frac{\bar{k} - \bar{k}^*}{(k_a - \bar{k})(k_a - \bar{k}^*)} \cdot \frac{f(k_a(\omega))}{x}.$$

Under assumption 1 this equation implies that as long as the oligopoly sector is more capital intensive than the competitive sector (viz., $k_a < k_o$), the ratio of competitive output to oligopoly output is higher (lower) in the home country if and only if $\bar{k}$ is lower (higher) than $\bar{k}^*$. Furthermore, if the preferences of the two countries are the same and homothetic, the expenditure ratio on each commodity is the same between the two countries. Therefore the standard HO result on the pattern of trade holds in this case. Formally,

Proposition 3 *If the preferences of the two countries are the same and homothetic, under assumption 1 each country exports the good that is relatively intensive in the country's relatively abundant factor. If the endowment ratio is the same across the two countries, the* *amount of trade is zero regardless the differences in the size of endowments and the level of ownership of firms.*

That is, goods trade takes place only on the traditional HO basis and Cournot oligopoly alone cannot form the basis of goods trade, whereas it can in Markusen (1981). From part 2 of proposition 2 and proposition 3, when we neutralize the traditional HO basis of trade by assuming that $\bar{k} = \bar{k}^*$, there is no trade of goods but there can be trade in firms (or FDI) depending on the relative endowments of firms.

Now we explore some properties of autarkic prices. It is well known that in the standard HO model the autarkic prices in the two countries are the same if the endowment ratios are the same in the two countries, i.e., size differences have no effect on the autarkic prices. Under the same condition there is no actual trade under free trade in the traditional HO model. As far as trade is concerned, a similar property holds in the present framework. From propositions 2 and 3, if $\bar{n}/\bar{n}^* = \bar{L}/\bar{L}^* = \bar{K}/\bar{K}^*$, there is neither goods trade nor FDI under free trade. But, will the autarkic prices in the two countries be the same? This is the question that we shall address now. For this, we denote by γ the size of the home country, i.e., the endowments in the home country are given by $\gamma\bar{L}$, $\gamma\bar{K}$ and $\gamma\bar{n}$.

For the subsequent analysis we need to specify homothetic preference as a special case of (9). Under autarky the demand function is

$$D = \gamma\bar{n}x = \phi(p)Y \quad \text{where} \quad \phi'(p) < 0. \tag{16}$$

With the above demand function, the first order profit maximizing condition (10) reduces to

$$\frac{\partial\pi}{\partial x} = \frac{[p - c(w(r), r)]\phi'(p)Y + x}{\phi'(p)Y} = 0. \tag{17}$$

Under demand function (16) a partial derivative of (17) with respect to another oligopolist's output y is

$$\frac{\partial^2\pi}{\partial x \partial y} = \frac{[\phi' + (p - c)\phi'']Y}{[\phi'(p)]^2}.$$

For strategic susbstitutability this value must be negative and hence

$$\phi' + (p - c)\phi'' < 0.$$

We also assume that the solution to the output problem exists for all values of $\bar{n}$ and $\bar{n}^*$ and for both free trade and autarky. Thus, in particular, when $\bar{n} = 1$, the autarkic equilibrium in the home country is characterized by a monopoly in the oligopoly sector, and we know that for a monopoly solution to exist the price elasticity of demand must be greater than unity. These assumptions are formally stated below.

Assumption 2 *$\epsilon > 1$, where ϵ $(= -\phi' p/\phi)$ is the price elasticity of demand for the oligopoly good, and the oligopoly firms are strategic substitutes in production, i.e., $\phi' + (p - c)\phi'' < 0$.*

We are now in a position to state and prove the result on autarkic prices (a formal proof is given in appendix A).

Proposition 4 *If $\bar{L}/\bar{L}^* = \bar{K}/\bar{K}^* = \bar{n}/\bar{n}^*$ and the preferences are same and homothetic and satisfy assumption 2, the autarkic price of the oligopoly good is lower in the larger country than in the smaller country. The rental rate of capital is higher (lower) in the larger country than in the smaller country if the oligopoly sector is more (less) capital intensive.*

This proposition implies that the country size does affect the autarkic goods and factor prices in the present context, as opposed to the standard HO model and Markusen (1981) with Cournot duopoly.

3.3 Gains from Trade

In this subsection we continue to neutralize the HO basis for trade and assume that

$$\bar{K}^* = \gamma^* \bar{K}, \ \bar{L}^* = \gamma^* \bar{L}, \ \bar{n}^* = \gamma^* \bar{n}, \tag{18}$$

where γ^* is the size of the foreign country relative to that of the home country. Under (18) propositions 2 and 3 imply

$$nx = D, \quad n = \bar{n}. \tag{19}$$

We can now state and prove a lemma which will be used to prove the main result in this section. A formal proof of the lemma is given in appendix B.

Lemma 2 *Under condition (18) a country is better off under free trade than under autarky if and only if the free trade output level of each oligopoly firm located in that country is higher than that under autarky.*

This result is related to a result in Helpman (1984, p.353) which derives a *sufficient* condition for a country to gain from trade in the presence of imperfect competition, and the condition is such that, in aggregate, the free-trade output levels of the imperfectly competitive sectors are higher than their respective autarkic levels. The intuition is simple. If free trade reduces oligopoly distortion, it must be welfare improving. In our special case with the possibility of FDI the condition turns out to be both *necessary* and *sufficient*.

The next obvious question to ask is: is the free-trade output level of each oligopoly firm higher than the autarkic level? If it is true, from lemma 2 free trade must be welfare superior to autarky. The following proposition states that the answer is unambiguously yes (see appendix C for the proof).

Proposition 5 *If $\bar{L}/\bar{L}^* = \bar{K}/\bar{K}^* = \bar{n}/\bar{n}^*$ and the preferences are same and homothetic, free trade is welfare superior to autarky.*

In the standard HO model, both countries benefit from improved terms of trade due to free trade and therefore gain from trade. In Markusen (1981) where there are just two firms — one in each country — and they are immobile, the firm in the larger country can make lower profits under free trade than under autarky. This loss of profits can outweigh gains from the terms-of-trade improvement, and thus the larger country can lose from trade. In Lahiri and Ono (1995) firms do not make any pure profits due to free entry and exit and thus both countries benefit from increased consumers' surplus due to increased competition in the industry after free trade. In the present paper, as in Markusen (1981), firms do make positive pure profits. However, unlike in Markusen (1981), firms are free to move between countries in order to realize higher profits. This mobility eliminates the possibility of firms losing profits under free trade, and thus both countries always gain from trade.

4 Comparative Statics

In this section we examine the effects of two parameters on the incidence of FDI. The parameters we consider are one from the production side and one from the consumption side. Equal changes in the parameters are considered in order to maintain two basic assumptions of an HO model, viz., identical technologies and preferences. The production-side parameter is the level of Hicks-neutral technological progress in the competitive sector of the two countries, given in (1). For the consumption side we consider a change in the relative demand for the oligopoly good in the two sectors.

4.1 Technological Progress

In this subsection we analyze the effect on the FDI level of technological progress in the competitive sectors of the two countries.

Totally differentiating the first equation of (5) and using (3) yields

$$\frac{dr}{r} = \frac{1}{\theta} \cdot d\theta - \frac{1}{\omega + k_a} \cdot d\omega, \quad \frac{dw}{r} = \frac{\omega}{\theta} \cdot d\theta + \frac{k_a}{\omega + k_a} \cdot d\omega. \tag{20}$$

Since $r = g'(k_o)$, totally differentiating the first equation of (7) gives

$$g\ dc = (\omega + k_o)\ dr + r\ d\omega. \tag{21}$$

By assuming homothetic preferences as in (16) in both countries and using lemma 1 we obtain

$$X = nx + n^*x^* = Nx = \phi(p)(Y + Y^*), \tag{22}$$

where from (8), (12) and lemma 1

$$Y + Y^* = w(\bar{L} + \bar{L}^*) + r(\bar{K} + \bar{K}^*) + N(p - c)x. \tag{23}$$

Under demand function (22) p_X $(= \partial p/\partial X) = 1/[(Y + Y^*)\phi']$. Substituting this into the first equation of (10) and using (22) leads to

$$-N(p - c)\phi'(p) = \phi(p). \tag{24}$$

From (22), (23) and (24) we obtain

$$w(\bar{L} + \bar{L}^*) + r(\bar{K} + \bar{K}^*) = x\left[\frac{N}{\phi} + \frac{\phi}{\phi'}\right]. \tag{25}$$

Substituting L_o obtained from the first and third equation of (13) into the first equation of (2) and applying n in (15) to the result yields

$$N(k_a - k_o)x = -g(k_o)\left[\bar{L}(\bar{k} - k_a) + \bar{L}^*(\bar{k}^* - k_a)\right]. \tag{26}$$

From (20), (21) and the total differentiations of (24), (25) and (26) we can derive the effects of a change in θ on p, ω and x. Once we solve $d\omega/d\theta$, the solution for $dn/d\theta$ can be solved by differentiating (15), which gives

$$N\bar{L}(\bar{k} - k_a)^2\ dn = n^2\bar{L}^*k_a'(\bar{k} - \bar{k}^*)\ d\omega. \tag{27}$$

It tells us that an increase in ω will result in an inflow of FDI to the home country if and only if the home country is relatively more capital abundant (viz. $\bar{k} > \bar{k}^*$). This is because an increase in ω enhances the home country's comparative advantage in the market for FDI.

Finally, after some manipulations and following the process outlined above, the solution for $d\omega/d\theta$ is solved as

$$\Delta_1 \; d\omega = \frac{A(k_a - k_o)}{\theta} \; d\theta, \tag{28}$$

$$\begin{aligned} \text{where} \;\; \Delta_1 &= \frac{[(\omega + k_a)L_o^W g' k_o' + g L_a^W k_a'](N\phi' + \phi^2)}{Nr\phi\phi'} - \frac{L_o^W N(\phi')^2(k_a - k_o)^2(\phi' + \phi^2)}{\phi^2(\omega + k_a)[(N+1)(\phi')^2 - \phi\phi'']}, \\ A &= (\omega + k_a)L_a^W + \frac{L_o^W(\omega + k_o)N(\phi')^2(\phi' + \phi^2)}{\phi^2[(N+1)(\phi')^2 - \phi\phi'']}, \\ L_o^W &= L_o + L_o^*, \quad L_a^W = L_a + L_a^*. \end{aligned}$$

Since (25) implies that $\phi' + \phi^2 < 0$, under assumption 2 $\Delta_1 > 0$.[6] The sign of A is however ambiguous since the first term is positive but the second one is negative. Therefore, combining (27) and (28) leads to

$$\frac{dn}{d\theta} \gtreqless 0 \qquad \text{according as} \qquad (k_a - k_o)(\bar{k} - \bar{k}^*)A \gtreqless 0.$$

The above result is best explained with an illustration. Suppose that the home country is Japan and that the foreign country is China. It is reasonable to assume then that Japan is more capital abundant (relative to labor) than China. Also, since the total size of population is much higher in China than in Japan and a large proportion of the Chinese workforce are employed in the agricultural sector, it is once again reasonable to assume that $L_a^W >> L_o^W$. The latter assumption implies that $A > 0$. It then follows that

$$\frac{dn}{d\theta} \gtreqless 0 \qquad \text{according as} \qquad (k_a - k_o) \gtreqless 0.$$

That is, an increase in θ will increase (decrease) the flow of FDI into China if the oligopoly sector is more capital (labor) intensive. This result is stated formally as a proposition.

[6] $\Delta_1 > 0$ also guarantees the Walrasian stability in the factor markets.

Proposition 6 *Suppose that the home country is relatively more capital abundant than the foreign country and that the world employment share of the oligopoly sector is small. Then, an equal technological progress in the competitive sectors of the two countries increases the FDI flow of the oligopoly sector into the foreign country if and only if the oligopoly sector is more capital intensive.*

The above result can intuitively be explained as follows. Suppose that the oligopoly sector is more capital intensive. Let us first examine how total oligopoly output Nx is affected by θ. Since Nx equals $\phi(p)(Y + Y^*)$ in equilibrium, there are two effects on it: an income effect via changes in $Y + Y^*$ and a price effect via changes in p. An increase in θ increases income in the agricultural sector and reduces that in the oligopoly sector, but the net effect is positive. An increase in θ also lowers ω (see equation (28) for the case where $k_a < k_o$) and thus increases the marginal cost in the oligopoly sector, which raises oligopoly price p and reduces output. Thus, the income effect on Nx is positive and the price effect is negative. However, if the world size of the agricultural sector is much larger than that of the oligopoly sector ($L_a^W >> L_o^W$), the positive income effect dominates the negative price effect.

Having found that Nx increases with θ, we shall explain how this increase coupled with relative factor endowments determine the effect on FDI. If the home country is more capital abundant, labor employment of the competitive (oligopoly) sector is smaller than that of the other sector in the home (foreign) country, implying that there is a smaller room for labor to move to the oligopoly sector in the home country. Thus, the increase in the oligopoly output is larger in the foreign country than in the home country, causing FDI to flow from the home country to the foreign country.

4.2 Change in Relative Preferences

In this subsection we examine the effect of a shift in preference for the oligopoly good in both countries on the incidence of FDI. Given that the oligopoly demand functions are

$$D = \lambda\phi(p)Y, \quad \text{and} \quad D^* = \lambda\phi(p)Y^*,$$

where λ represents the parameter of the preference shift. Under them $p_X = 1/[(Y + Y^*)\lambda\phi']$ and then equations (24), (26) and (27) from the previous subsection are still valid. Only equation (25) changes to

$$w(\bar{L} + \bar{L}^*) + r(\bar{K} + \bar{K}^*) = x\left[\frac{N}{\lambda\phi} + \frac{\phi}{\phi'}\right]. \tag{29}$$

Following similar calculations as in the preceding subsection, we get

$$\Delta_2 \; d\omega = \frac{Nx(k_a - k_o)}{\lambda^2\phi} \cdot d\lambda, \tag{30}$$

where from (25) and assumption 2 Δ_2 satisfies

$$\Delta_2 = -\frac{NrL_o^W(k_a - k_o)^2(\phi')^2(\phi' + \phi^2)}{(\omega + k_a)[(N+1)(\phi')^2 - \phi\phi'']} + \frac{g[(\omega + k_a)L_o k_o' + (\omega + k_o)L_a^W k_a'](N\phi' + \phi^2)}{N(\omega + k_o)\phi\phi'} > 0.$$

$$\text{Therefore,} \qquad \frac{d\omega}{d\lambda} \gtreqless 0 \qquad \text{according as} \qquad k_a \gtreqless k_o.$$

A rise in λ increases demand and hence expands output in the oligopoly sector. If the oligopoly good is more capital intensive ($k_a < k_o$), it increases capital demand relative to labor demand, which lowers wage-rental ratio ω.

Finally, substituting (30) into (27) yields

$$\frac{dn}{d\lambda} = \frac{n^2 x\bar{L}^* k_a'(k_a - k_o)(\bar{k} - \bar{k}^*)}{\bar{L}(\bar{k} - k_a)^2\Delta_2\lambda^2\phi},$$

from which we derive the following proposition.

Proposition 7 *Suppose that the home country is relatively more capital abundant. Then, an upward shift in the preference for the oligopoly good in the two countries will increase the flow of FDI into the labor abundant foreign country if and only if the oligopoly sector is more capital intensive.*

Intuitively, an increase in λ results in a shrinkage of the competitive sector, releasing resources for the oligopoly sector and expanding it in both countries. However, since the foreign country is relatively more labor abundant and the competitive sector is relatively larger, more resources are released from the competitive sector in the foreign country than in the home country. Therefore, the oligopoly sector expands more in the labor abundant country than in a capital abundant one. This in turn leads to an outflow of oligopoly firms from the capital abundant country to the labor abundant one.

5 Conclusion

In this paper we develop a two-country, two-good, two-factor model of international trade where one of the two sectors of production is oligopolistic and each country is endowed with a fixed number of firms belonging to that sector. That is, apart from the endowments of labor and capital, the two countries are also 'endowed' with a number of firms in the oligopoly sector. The firms are however mobile between the two countries and the foreign direct investment (FDI) equilibrium is achieved by equating the profit levels in the two countries. Changes in factor prices play a crucial role in obtaining this equilibrium.

Under the above-mentioned framework, we first of all analyze some of the time-honored issues in trade theory, viz., the issue of factor price equalization, that of patter of trade, and finally the question of gains from trade. In our framework we also have the additional issue of the pattern of FDI.

There is now a small literature which examines the above issues in oligopolistic HO models with different characteristics of the oligopoly sector. When there are only two firms — one in each country — it has been proved that some of the traditional results of the competitive HO model, viz., factor price equalization under free trade, gains from trade etc., fails to go through under imperfect competition. Moreover, international duopoly can be a basis for trade; that is, when the factor endowment ratios are the same between the two countries, international trade can still take place under free trade. It has also been shown in the literature that if, instead of a fixed number of firms in each country, free entry and exit of firms are allowed in each country, the traditional results are reestablished and imperfect competition cannot be a basis for trade. However, no one in the literature has allowed for free mobility of firms between the two countries which we do here.

As for the conventional issues, our findings are that most of the results from the traditional HO model go through in the present framework. Thus, there are two ways of reestablishing the traditional HO results in oligopolistic HO models. The first is to allow for free entry and exit of firms in each country, and the second is to allow for free mobility of a given number of firms between the two countries. In addition, we also derive conditions in terms of relative endowment ratios to obtain a particular pattern of FDI in this paper.

The second purpose of this paper is to examine the effect of changes in two exogenous parameters on the incidence of FDI. We choose on a parameter from the production side, viz., technological progress in the competitive sector, and a parameter from the consumption side, viz., a shift parameter representing preference for the oligopoly good. In this exercise we identify the roles of relative factor intensities and relative factor endowments on the results.

Appendix A: Proof of proposition 4

Substituting Y in (16) to (17) gives

$$\gamma\bar{n}(p-c)\phi'(p)+\phi(p)=0. \tag{A.1}$$

Totally differentiating (A.1) and using (14) leads to

$$[(1+\gamma\bar{n})\phi'+\gamma\bar{n}(p-c)\phi'']dp+\frac{\gamma\bar{n}\phi'(k_a-k_o)}{g}\,dr=\frac{\phi}{\gamma}\,d\gamma. \tag{A.2}$$

From (2) and the first and the third equation of (13) we derive

$$\frac{\bar{L}(\bar{k}-k_a)}{(k_o-k_a)\bar{n}}=L_o=\frac{x}{g(k_o)}. \tag{A.3}$$

Using (8) and (16), we derive

$$Y=\gamma\bar{L}(w+r\bar{k})+\gamma\bar{n}\pi=\gamma\bar{L}(w+r\bar{k})+(p-c)\phi(p)Y,$$

and thus,

$$Y=\frac{\gamma\bar{L}(w+r\bar{k})}{1-(p-c)\phi(p)}. \tag{A.4}$$

Substituting x from (16) and Y from (A.4) into (A.3) leads to

$$\left[\frac{1}{\phi(p)}-(p-c)\right](\bar{k}-k_a)g(k_o)-(w+r\bar{k})(k_o-k_a)=0. \tag{A.5}$$

The left hand side of (A.5) is the excess demand for capital in the country. Totally differentiating this equation yields

$$(\omega+k_a)(\omega+\bar{k})\left(\frac{\bar{k}-k_o}{\bar{k}-k_a}k_a'-\frac{\omega+k_a}{\omega+k_o}k_o'\right)dr=g(1+\frac{\phi'}{\phi^2})(k_a-\bar{k})dp. \tag{A.6}$$

Since (16) gives

$$1+\frac{\phi'}{\phi^2}=\frac{1}{p\phi}\cdot\left[\frac{pD}{Y}-\epsilon\right], \tag{A.7}$$

where $\epsilon(= -\phi' p/\phi) > 1$ from assumption 2 and pD/Y, the share of the oligopoly good in national income, is always less than unity, the right hand side of (A.7) is always negative and thus

$$1 + \frac{\phi'}{\phi^2} < 0. \tag{A.8}$$

Therefore, from (4), assumption 1 and (A.6) we derive

$$\frac{dp}{dr} \gtreqless 0 \quad \text{according as} \quad k_a \gtreqless \bar{k}. \tag{A.9}$$

By substituting dp from (A.2) into (A.6) we obtain

$$\Delta \frac{dr}{d\gamma} = \frac{\phi g(k_a - \bar{k})\left(1 + \frac{\phi'}{\phi^2}\right)}{\gamma[(1+\gamma\bar{n})\phi' + \gamma\bar{n}(p-c)\phi'']}, \tag{A.10}$$

$$\text{where } \Delta = (\omega + k_a)(\omega + \bar{k})\left(\frac{\bar{k} - k_o}{\bar{k} - k_a}k_a' - \frac{\omega + k_a}{\omega + k_o}k_o'\right) + \frac{\gamma\bar{n}\phi'(k_a - k_o)(k_a - \bar{k})\left(1 + \frac{\phi'}{\phi^2}\right)}{(1+\gamma\bar{n})\phi' + \gamma\bar{n}(p-c)\phi''},$$

and Δ is shown to be negative by (4), assumptions 1 and 2, and (A.8). Note that Δ is the slope of the excess demand function for capital, and thus this property guarantees the system to be Walrasian stable. Therefore, from assumption 2, (A.8), (A.9) and (A.10) we obtain

$$\frac{dr}{d\gamma} \gtreqless 0 \text{ according as } \bar{k} \gtreqless k_a,$$
$$\frac{dp}{d\gamma} < 0.$$

$\square$

Appendix B: Proof of lemma 2

We prove this result using the principle of Revealed Preference.[7]

[7] See, for example, Markusen (1981), Markusen and Melvin (1984) and Lahiri and Ono (1989) for the application of the Revealed Preference Principle in international trade theory.

Total value of outputs evaluated at marginal cost level (which is the shadow price) is maximized subject to the production possibility frontier if the frontier is concave. Therefore under free trade and under autarky, we get respectively the following properties:

$$A + ncx \geq A^a + \bar{n}cx^a, \tag{B.1}$$

$$A^a + \bar{n}c^a x^a \geq A + nc^a x, \tag{B.2}$$

where superscript a denotes the equilibrium values under autarky. The condition for balance for payments under free trade, (19) and the condition of autarky give

$$D_A = A, \quad D^a = \bar{n}x^a, \quad D_A^a = A^a, \tag{B.3}$$

where D_A is the demand for the numeraire good.

Since the bundle $\{D_A, D\}$ and $\{D_A^a, D^a\}$ are both feasible under the price set $\{1, p\}$, $\{D_A, D\}$ is always preferred to $\{D_A^a, D^a\}$. Therefore, it can be shown that the free trade equilibrium is preferred to the autarkic one if

$$D_A + pD \geq D_A^a + pD^a. \tag{B.4}$$

Similarly, the autarkic equilibrium is preferred to the free trade equilibrium if

$$D_A^a + p^a D^a \geq D_A + p^a D. \tag{B.5}$$

Substituting D in (19) and D_A, D^a and D_A^a in (B.3) into (B.4) and using (B.1) and the second equation of (19) yields

$$D_A + pD - (D_A^a + pD^a) \geq (p - c)\bar{n}(x - x^a).$$

This property implies that free trade is preferred to autarky if $x \geq x^a$. Furthermore, substituting D in (19) and D_A, D^a and D_A^a in (B.3) into (B.5) and using (B.1) and the second equation of (19) yields

$$D_A^a + p^a D^a - (D_A + p^a D) \geq \bar{n}(p^a - c^a)(x^a - x),$$

which implies that autarky is preferred to free trade if $x^a \geq x$. Thus, free trade is preferred to if and only if $x \geq x^a$. $\square$

Appendix C: Proof of proposition 5

Note that when $\gamma^* = 0$, the equilibrium for the home country is the autarkic one. Thus, if we show that $dx/d\gamma^* > 0$, then using lemma 2 we prove the result for the home country. Exactly analogous analysis can then be made for the foreign country.

Since there is no trade and FDI under free trade (propositions 2 and 3), the demand function is given by

$$D = \bar{n}x = \phi(p)Y, \tag{C.1}$$

where $Y = w(r)\bar{L} + r\bar{K} + \bar{n}\pi$. Differentiating (A.3) and using the first equation of (3) and (6), we find

$$\bar{n}r(k_o - k_a)^2 \, dx = g'(\omega + k_a)\bar{L}[(\bar{k} - k_a)k_o'(\omega + k_a) - k_a'(\omega + k_o)(\bar{k} - k_o)] \, dr. \tag{C.2}$$

For the determination of r, equation (A.5) will continue to hold here, and equation (A.1) is modified with γ replaced by $1+\gamma^*$. This is because in view of proposition 1 and condition (18), we get the world demand as $D + D^* = (1+\gamma^*)\phi(p)Y$. Therefore, equation (A.10) and proposition 4 are still valid. Using this result in (C.2) and making use of assumption 1 and lemma 2 we have the result. $\square$

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