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Endogenous Leadership with and without Policy Intervention: International Trade when Producer and Seller Differ

By

Zhifang Peng and Sajal Lahiri

Department of Economics
Southern Illinois University Carbondale
Carbondale, U.S.A.
(E-mails: faye04@gmail.com and lahiri@siu.edu)

Abstract

Using a vertically linked model of international trade where producers and sellers are different entities and belong to two different countries, we examine the issue of endogenous leadership. In the absence of policy intervention, there are two cases depending on whether the producer or the seller is the leader. In the presence of policy intervention, the nationality of the leader and that of the follower also becomes important. We find necessary and sufficient conditions for endogenous leadership to arise, and find that in the presence of policy intervention and lump-sum transfers, leadership by the domestic firm — whether it is a producer or a seller — will emerge as the equilibrium.

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Mailing Address: Zhifang Peng, Department of Economics, Southern Illinois University Carbondale, MC 4515, 1000 Faner Drive, Carbondale, IL. 62901, U.S.A; FAX: +1-618-453-2717; Email: faye04@gmail.com

1 Introduction

International inter-firm collaboration has been growing hand in hand with the overall process of integration of the international economy. Such collaborations have been attracting the attention of scholars from many fields including economics (see, for example, Backman, 1965; Berg and Friedman, 1977; Duncan, 1982; McConnell and Nantell, 1985)). However, bulk of this literature in the field of international business organization concerns themselves primarily with the situation where both domestic and foreign firms have complementary specific inputs and join together to produce a product, sell the product, and share in the resulting profits and losses. However, sellers are commonly distinct from producers in real life. For example, in China, Shanghai General Motors Co. Ltd produce automobiles, sells them to General Motors Warehousing and Trading (Shanghai), a wholly-owned GM enterprise which in turn sells automobiles at home and at abroad.

In spite of its prevalence in real life, the study of vertical inter-firm collaboration is relatively uncommon in the literature of international trade. Nearly eighty years ago, Bowley (1928) studied the implications of vertical relationships, and presented the first formal statement of the price leadership problem. subsequently, others considered a joint-profit-maximization model to solve the problem of determinacy of the quantity of intermediate inputs traded (see, for example, Fellner,1947; Machlup and Taber, 1960; Truett and Truett, 1993). Henderson and Quandt (1980) analyze four possibilities that arise in the determination of equilibrium in a bilateral monopoly model, viz., (i) seller is the leader, the producer take the price as given, (ii) producer is the leader, the seller take the price as given, (iii) collusion by the seller and the buyer, and (iv) noncooperation by the seller and buyer in which both of them may go out of business. Other studies that provide game theoretic solution to the problem include Devadoss and Cooper (2000) where the two entities have equal bargaining power, and Dasgupta and Devadoss (2002) where the two parties have unequal bargaining power. Lahiri and Ono (1999) put the bilateral monopoly structure in the international context and examine the question of optimal tariffs under different market structures.

In the studies discussed above, the issue of leadership, whenever applied, are exogenous in nature; that is, who is the leader and who is follower is not determined endogenously inside the model. There is however a separate literature in which the issue of leadership is determined endogenously under different contexts. Ono (1978; 1982), Hamilton and Slutsky (1990), Matsumura (1995, 2003) examine the issue in a domestic context under different market structures. Das and Lahiri (2006) develops a model of terrorist activity and examines the issue of endogenous leadership between a terrorist organization and a government trying to deal with potential terrorist attacks. Jafarey and Lahiri (2007) considers interactions between a borrowing country and a monopolist lender, and analyzes if one the two parties can emerge as the leader.

In the present paper, we integrate the two different literature discussed above. To be more specific, we shall consider a model in which there is a producer and a seller who are engaged in a vertical relationship. We shall examine if the leadership by one of the parties can be agreed upon by both parties, considering their own self interests. We provide an international dimension to the problem by assuming that the two parties are national of two different countries, called the home and the foreign country. We also consider two situations depending on whether the domestic government is passive or it is active in the sense of optimally setting tax/subsidy instruments. In the latter situation, the ownership of the two different parties becomes particularly relevant. In this situation, it also matters as to which of the two is a producer and which one is the seller.

The following section considers the case where the domestic government is passive and does not intervene at all. In section 3 , we consider the optimal tax/subsidy policy of the domestic government. This section is divided into a number of subsections depending who is the producer and who is the seller, and in each case who is the leader and who is the follower. Some concluding remarks are made in section 4.

2 The Model with no Government Intervention

We consider a single homogeneous-commodity partial-equilibrium model with two entities, a producer and a seller. The government is passive in this section so that the nationality of the two parties are not relevant for our analysis here. We model a simple vertical relationship between these two entities.

The inverse demand function for the commodity is given by:

$$p = a - bx. \tag{1}$$

where p and x are respectively the price and demand/output of the commodity, and a and b are positive parameters. Since we do not consider government behavior, it does not matter, for the purpose of this section, where this good is consumed.

The cost functions for the producer and that of the seller are respectively:

$$c^p = \beta^p x + \theta^p x^2, \tag{2}$$

$$c^s = \beta^s x + \theta^s x^2, \tag{3}$$

where the β's and the θ's are positive parameters. Whereas c^p represents purely production costs, c^s includes costs associated with retailing, marketing etc.

The profit of the producer and that of the seller respectively are:

$$\pi^p = \rho x - \beta^p x - \theta^p x^2, \tag{4}$$

$$\pi^s = (a - bx - \rho - \beta^s - \theta^s x)x, \tag{5}$$

where ρ is the inside price of the commodity, i.e., the price which the producer receives from the seller.

Before proceeding any further, i.e. before specifying profit-maximizing conditions, we need to consider two scenarios depending who the leader is, and these are taken up in the following two subsections.

2.1 The case of leadership by the seller

Using the model developed so far, we now derive the profit maximization conditions when the seller is the leader. We consider a following two stage game, working with backward induction. When the seller acts as a leader, the producer, in the second stage, takes the inside price ρ as given, and maximizes its profits given by (**??**) yielding the first order condition:

$$\frac{\partial \pi^p}{\partial x} = \rho - \beta^p - 2\theta^p x = 0,$$

which gives the reaction function of the producer:

$$\rho(x) = \beta^p + 2\theta^p x. \tag{6}$$

Since the seller is the leader and moves first, it maximizes its profits π^s given in (**??**), taking into account the producer's reaction function given by (**??**). The first-order profit-maximizing condition is given by:

$$\begin{aligned} \frac{\partial \pi^s}{\partial x} &= \frac{\partial[((a-\beta^s) - (b+\theta^s)x - \beta^p - 2\theta^p x)x]}{\partial x} \\ &= (a-\beta^s) - (b+\theta^s)x - \beta^p - 2\theta^p x - (b+\theta^s)x - 2\theta^p x = 0. \end{aligned} \tag{7}$$

From (**??**) and (**??**), output x and inside price ρ are calculated, respectively, as:

$$x_{s1} = \frac{a - \beta^s - \beta^p}{2(b+\theta^s+2\theta^p)}, \tag{8}$$

$$\rho_{s1} = \frac{(a-\beta^s+\beta^p)\theta^p + (b+\theta^s)\beta^p}{b+\theta^s+2\theta^p}, \tag{9}$$

and then profits of the producer and that of the seller are calculated, respectively, as:

$$\pi_2^p = \frac{(a-\beta^s-\beta^p)^2\theta^p}{4(b+\theta^s+2\theta^p)^2}, \tag{10}$$

$$\pi_1^s = \frac{(a-\beta^s-\beta^p)^2}{4(b+\theta^s+2\theta^p)}. \tag{11}$$

This completes the analysis when the seller is the leader, and we now turn to the case where the producer is the leader.

2.2 The case of leadership by the producer

When the producer is the leader, the seller takes the inside price ρ as given, and hence its first-order profit-maximizing condition is:

$$\frac{\partial \pi^s}{\partial x} = a - bx - \rho - \beta^s - \theta^s x - bx - \theta^s x = 0,$$

which gives the reaction function of the seller as:

$$\rho(x) = a - \beta^s - 2(b + \theta^s)x. \tag{12}$$

Since the producer is the leader, it maximizes π^p given by (**??**), taking into account the seller's reaction function given by (**??**). The profit-maximizing condition is given by:

$$\begin{aligned}\frac{\partial \pi^p}{\partial x} &= \frac{\partial[((a - \beta^s) - 2(b + \theta^s)x - \beta^p - \theta^p x)x]}{\partial x} \\ &= (a - \beta^s) - 2(b + \theta^s)x - \beta^p - \theta^p x - 2(b + \theta^s)x - \theta^p x = 0. \end{aligned} \tag{13}$$

The output x and the inside price ρ are calculated, respectively, as:

$$x_{s2} = \frac{a - \beta^s - \beta^p}{2(2b + 2\theta^s + \theta^p)}, \tag{14}$$

$$\rho_{s2} = \frac{(a - \beta^s + \beta^p)(b + \theta^s) + a\theta^p}{2b + 2\theta^s + \theta^p}. \tag{15}$$

The profit of the producer and that of the seller are:

$$\pi_1^p = \frac{(a - \beta^s - \beta^p)^2}{4(2b + 2\theta^s + \theta^p)}, \tag{16}$$

$$\pi_2^s = \frac{(a - \beta^s - \beta^p)^2(b + \theta^s)}{4(2b + 2\theta^s + \theta^p)^2}. \tag{17}$$

2.3 Leadership preferences

Having derived the levels of profits for the two firms under different leadership scenario, we can now examine if one of the scenarios will be accepted by both parties. comparing the profits of the two firms under the two scenarios we find:

$$\pi_1^p - \pi_2^p = \frac{(a-\beta^s-\beta^p)^2((b+\theta^s)^2 - 2(b+\theta^s)\theta^p + 3(\theta^p)^2)}{4} > 0,$$

$$\pi_1^s - \pi_2^s = \frac{(a-\beta^s-\beta^p)^2(3(b+\theta^s)^2 + 2(b+\theta^s)\theta^p + (\theta^p)^2)}{4(b+\theta^s+2\theta^p)(2b+2\theta^s+\theta^p)^2} > 0.$$

That is, each firm prefers its own leadership. In other words, in the absence of any side payments, there will be unanimity about who should be the leader. Formally,

Proposition 1 *In the absence of side payment, both entities will prefer their own leadership, and unanimity cannot be reached.*

Thus, in the absence of side payments, the relationship between the two firms may break down. However, if side payments are possible, then we only need to look at the differences in the aggregate profits of the two firms, and if the aggregate profit is higher under one scenario, that scenario can be acceptable to both parties and should be the equilibrium outcome.

The levels of, and the difference between, aggregate profits when the seller is the leader and that when the producer is the leader are calculated as:

$$\pi_{s1} = \pi_2^p + \pi_1^s = \frac{(a-\beta^s-\beta^p)^2(b+\theta^s+3\theta^p)}{4(b+\theta^s+2\theta^p)^2},$$

$$\pi_{p1} = \pi_1^p + \pi_2^s = \frac{(a-\beta^s-\beta^p)^2(3b+3\theta^s+\theta^p)}{4(2b+2\theta^s+\theta^p)^2},$$

$$\pi_{s1} - \pi_{p1} = \frac{[(a-\beta^s-\beta^p)^2(b+\theta^s)^2 + (b+\theta^s)\theta^p + (\theta^p)^2 + 3(b+\theta^s)\theta^p)](b+\theta^s-\theta^p)}{4(b+\theta^s+2\theta^p)^2(2b+2\theta^s+\theta^p)^2}.$$

From the last of the above three equations it follows that:

$$\pi_{s1} - \pi_{p1} > 0 \quad \Longleftrightarrow \quad b+\theta^s > \theta^p.$$

Proposition 2 *With the presence of side payment, both firms will prefer seller's (producer's) leadership if and only if $b+\theta^s > \theta^p$ ($b+\theta^s < \theta^p$)*

The intuition for the above result is as follows. Note that when $\theta^p \simeq 0$, the leadership by the seller is preferred by both. In this case, the marginal cost of production is effectively constant, and the equilibrium inside price is nearly equal to the marginal cost of production. That is, production is almost at the most efficient level, and the total unit cost for the seller is almost at the highest possible level. Not surprisingly therefore, under the leadership of the seller, efficiency level of operation and thus total surplus is very high. As the marginal cost of production increase with output level we move more and more away from the efficiency of the leadership by the seller and after some stage total surplus becomes higher when the producer is the leader.

3 The model with government intervention

In the previous section, we did not consider government policy intervention, and therefore the nationality of the two firms were of no concern to us. In this section, we add the government as the third player who commits on a specific sales tax t at the first stage of the game, the subsequent two stages of the game are the same as before. When the domestic firm is the seller and the foreign firm the producer (subsection **??** below), the tax t is the consumption tax, and when foreign firm is the seller and the domestic is the producer (subsection **??**), the tax t is an exports tax.

The profits of the producer and that of the seller respectively are:

$$\begin{aligned} \pi^p &= \rho x - \beta^p x - \theta^p x^2, \\ \pi^s &= (a - bx - \rho - \beta^s - \theta^s x - t)x \end{aligned}$$

In the following two subsection, we shall consider two scenarios according to what the domestic firm does: (a) domestic firm is the seller (section **??**), and (b) domestic firm is the producer (section **??**). Each subsection will then be subdivided according to the types of producer-seller relationships: (i) leadership by seller, and (ii) leadership by producer.

3.1 Domestic firm is the seller

First we consider the case when the domestic firm is the seller and also the leader.

3.1.1 Domestic seller is the leader

In stage three of the game, foreign producer who is the follower, takes the inside price ρ as given. Its profit-maximizing condition, as before, is:

$$\frac{\partial \pi^f}{\partial x} = \rho - \beta^p - 2\theta^p x = 0,$$

giving rise to its reaction function:

$$\rho(x) = \beta^p + 2\theta^p x. \tag{18}$$

In stage two, the domestic seller, who is the leader, maximizes its profits, given by $\pi^d = (a - bx - \beta^p - 2\theta^p x - \beta^s - \theta^s x - t)x$, taking into account the producer's reaction function (**??**). The first-order profit-maximizing condition is:

$$\frac{\partial \pi^d}{\partial x} = a - 2bx - \beta^p - 4\theta^p x - \beta^s - 2\theta^s x - t = 0. \tag{19}$$

From (**??**), the solution of output/sales, x_{ds1}, and then from (**??**) the inside price ρ_{ds1} are given by:

$$x_{ds1} = \frac{a - \beta^s - \beta^p - t}{2(b + \theta^s + 2\theta^p)}, \tag{20}$$

$$\rho_{ds1} = \frac{(a - \beta^s + \beta^p - t)\theta^p + (b + \theta^s)\beta^p,}{b + \theta^s + 2\theta^p}.$$

For a given tax rate t, profits of the domestic firm and that of the foreign firm can be calculated

$$\pi^f_{p2} = \frac{(a - \beta^s - \beta^p - t)^2\theta^p}{4(b + \theta^s + 2\theta^p)^2}, \tag{21}$$

$$\pi^d_{s1} = \frac{(a - \beta^s - \beta^p - t)^2}{4(b + \theta^s + 2\theta^p)}. \tag{22}$$

In stage one the game, the domestic government chooses the tax rate t to maximize the welfare of the home country, W, which is the sum of consumers' surplus, CS, domestic seller's profits, π^d, and the government's tax revenue, tx, i.e.,

$$W = CS + \pi^d + tx, \tag{23}$$

where it is well known that $d\text{CS} = -xdp$.

Substituting π^d from (??) and x from (??) into (??) and then setting $\partial W/\partial t = 0$, we can solve the optimal tax rate t as:

$$t = -\frac{(a - \beta^s - \beta^p)b}{b + 2\theta^s + 4\theta^p} < 0. \tag{24}$$

Now, substituting (??) into (??) and (??), we get:

$$\pi^f_{p2} = \frac{(a - \beta^s - \beta^p)^2\theta^p}{(b + 2\theta^s + 4\theta^p)^2},$$

$$\pi^d_{s1} = \frac{(a - \beta^s - \beta^p)^2(b + \theta^s + 2\theta^p)}{(b + 2\theta^s + 4\theta^p)^2}.$$

3.1.2 Domestic seller is the follower

When the foreign producer is the leader and the domestic seller is the follower, in stage 3, the domestic firm's profit-maximizing behavior given rise to its reaction function as:

$$\rho(x) = a - \beta^s - 2(b + \theta^s)x - t, \tag{25}$$

Since the producer, who is the leader, maximizes π^f $(= ((a-\beta^s)-2(b+\theta^s)x-\beta^p-\theta^p x-t)x)$, taking into account the seller's reaction function (??), its profit-maximization behavior gives rise to the solutions of output x and then the inside price ρ in this case as:

$$x_{ds2} = \frac{a - \beta^s - \beta^p - t}{2(2b + 2\theta^s + \theta^p)},$$

$$\rho_{ds2} = \frac{(a - \beta^s + \beta^p - t)(b + \theta^s) + (a - t)\theta^p}{2b + 2\theta^s + \theta^p}$$

The profits of the producer and that of the seller in this case are:

$$\pi^f_{p1} = \frac{(a-\beta^s-\beta^p-t)^2}{4(2b+2\theta^s+\theta^p)}, \tag{26}$$

$$\pi^d_{s2} = \frac{(a-\beta^s-\beta^p-t)^2(b+\theta^s)}{4(2b+2\theta^s+\theta^p)^2}. \tag{27}$$

The first stage of the game is similar to the previous case here, and the optimal tax rate is given by:

$$t = \frac{(a-\beta^s-\beta^p)(b+2\theta^s+2\theta^p)}{5b+6\theta^s+4\theta^p} > 0. \tag{28}$$

Substituting (**??**) into (**??**) and (**??**), profits of the two firms in this case are given by:

$$\pi^d_{s2} = \frac{(a-\beta^s-\beta^p)^2(b+\theta^s)}{(5b+6\theta^s+4\theta^p)^2}, \tag{29}$$

$$\pi^f_{p1} = \frac{(a-\beta^s-\beta^p)^2(2b+2\theta^s+\theta^p)}{(5b+6\theta^s+4\theta^p)^2} \tag{30}$$

Note that while the optimal tax rate is positive in the case when the domestic seller is the follower, it is negative when the domestic seller is the leader.[1] The reason for the change is the sign of the optimal tax is that the domestic firms profits are higher when it is the leader, and therefore in the determination of the optimal tax rate, the interest of the domestic firm plays a bigger role when the domestic firm is the leader than when it is the follower. This results in a lower optimal tax (in fact a subsidy) when the domestic firm is a leader rather than a follower.

3.1.3 Leadership preference

In this section, we shall check If there is a certain producer-seller relationship that both firms prefer. Comparing profits of each firm under the two scenarios, we get:

$$\pi^f_{p1} - \pi^f_{p2} = \frac{(a-\beta^s-\beta^p)^2}{(5b+6\theta^s+4\theta^p)^2(b+2\theta^s+4\theta^p)^2}\Delta^f,$$

$$\pi^d_{s1} - \pi^d_{ds2} = \frac{(a-\beta^s-\beta^p)^2}{(5b+6\theta^s+4\theta^p)^2(b+2\theta^s+4\theta^p)^2}\Delta^d > 0$$

[1] It is to be noted that Lahiri and Ono (1999) also found that the sign of optimal tariffs can crucially depend on who is the leader.

where

$$\Delta^f = (2b + 2\theta^s + \theta^p)(b + 2\theta^s + 4\theta^p)^2 - \theta^p(5b + 6\theta^s + 4\theta^p)^2,$$

$$\Delta^d = [(b + \theta^s)((5b + 6\theta^s + 4\theta^p)^2 - (b + 2\theta^s + 4\theta^p)^2)] + 2\theta^p(5b + 6\theta^s + 4\theta^p)^2 > 0.$$

That is, the domestic firm would prefer to be the leader. Moreover, in this case, the foreign firm may also prefer the domestic seller's leadership to its own if $\theta^s < (4/5)\theta^p$ since $\Delta^f < 0$ under this condition. This result is stated formally in proposition **??** below.

Proposition 3 *In the absence of side payments, leadership by the domestic seller is an equilibrium if* $\theta^s < (4/5)\theta^p$.

The intuition is as follows. Since the domestic seller receives a higher subsidy when it is the leader, the vertically related foreign firm also benefits from this higher subsidy. This competes with the natural tendency for the foreign firm to prefer its own leadership. The net effect goes in favor of the foreign firm preferring the domestic firm's leadership when the foreign firm's marginal cost function is sufficiently steep, i.e., θ^p is sufficiently high.

The levels of, and the differences between, aggregate profits when the domestic seller is the leader and that when the foreign producer is the leader are:

$$\pi_{ds1} = \pi^d_{s1} + \pi^f_{p2} = \frac{(a - \beta^s - \beta^p)^2(b + \theta^s + 3\theta^p)}{(b + 2\theta^s + 4\theta^p)^2},$$

$$\pi_{ds2} = \pi^d_{s2} + \pi^f_{p1} = \frac{(a - \beta^s - \beta^p)^2(3b + 3\theta^s + \theta^p)}{(5b + 6\theta^s + 4\theta^p)^2},$$

$$\pi_{ds1} - \pi_{ds2} = \frac{(a - \beta^s - \beta^p)^2}{(5b + 6\theta^s + 4\theta^p)^2(b + 2\theta^s + 4\theta^p)^2}\Delta > 0,$$

where

$$\Delta = (b + \theta^s + 3\theta^p)(5b + 6\theta^s + 4\theta^p)^2 - (3b + 3\theta^s + \theta^p)(b + 2\theta^s + 4\theta^p)^2 > 0.$$

That is, when side payments are allowed, the foreign firm will unambiguously prefer leadership by the domestic firm, which will be the equilibrium outcome. Formally,

Proposition 4 *In the presence of side payments, when government optimally employs sales tax and when the domestic firm is the seller, both firms will prefer the domestic firms' leadership.*

A higher subsidy when the domestic firm is the leader, makes the scenario so much efficient in the sense of higher income that the aggregate profits becomes unambiguously higher in this case than in the case where the domestic firm is the follower.

3.2 Domestic firm is the producer

Finally, in this subsection, we consider the case when domestic firm is the producer, and the foreign firm is the seller. In this situation, it more reasonable to assume that the good is consumed outside the country and the sales tax t need to be reinterpreted as an exports tax. Once, we shall consider two scenarios here, depending on who is the leader, and these are taken up in the two subsections below.

3.2.1 Domestic producer is the leader

When the domestic producer acts as a leader, the foreign seller takes inside price ρ as given, and hence its profit maximization condition gives the following reaction function of the foreign seller:

$$\rho(x) = a - \beta^s - 2(b + \theta^s)x - t. \tag{31}$$

The producer, who is the leader, maximizes π^d $(= ((a-\beta^s)-2(b+\theta^s)x-\beta^p-\theta^p x-t)x)$by taking into account the seller's reaction function as given in (**??**). The first-order its profit-maximizing condition gives the solution of x and thus ρ from (**??**) as:

$$\begin{aligned} x_{dp1} &= \frac{a - \beta^s - \beta^p - t}{2(2b + 2\theta^s + \theta^p)}, \\ \rho_{dp1} &= \frac{(a - \beta^s + \beta^p - t)(b + \theta^s) + (a - t)\theta^p}{2b + 2\theta^s + \theta^p}. \end{aligned}$$

Profits of the domestic producer and that of the foreign seller are calculated, respectively, as:

$$\pi_{p1}^{d} = \frac{(a-\beta^s-\beta^p-t)^2}{4(2b+2\theta^s+\theta^p)}, \tag{32}$$

$$\pi_{s2}^{f} = \frac{(a-\beta^s-\beta^p-t)^2(b+\theta^s)}{4(2b+2\theta^s+\theta^p)^2}. \tag{33}$$

As before, in stage one, the domestic government chooses tax t to maximize the welfare of the home country, $\tilde{W}$, which is the sum of the domestic seller's profit, π^d, and the government's tax revenue, tx; that is:[2]

$$\tilde{W} = \pi^d + tx. \tag{34}$$

Substituting the expressions of π^d and x from above and then setting $d\tilde{W}/dt = 0$, we solve the optimal level of t as:

$$t = 0..$$

The negative effect of increasing t on profits cancels with the positive effect of increasing t on tax revenue (for a given level of x, leaving only the dead-weight loss, and thus the optimal export tax is zero.

By substituting the optimal values of t in the expression for profits in (**??**) and (**??**), we get:

$$\pi_{p1}^{d} = \frac{(a-\beta^s-\beta^p)^2}{4(2b+2\theta^s+\theta^p)},$$

$$\pi_{s2}^{f} = \frac{(a-\beta^s-\beta^p)^2(b+\theta^s)}{4(2b+2\theta^s+\theta^p)^2}.$$

3.2.2 Domestic producer is the follower

We now consider the second scenario where the domestic producer is the follower. Here, in stage three, the follower, i.e., the domestic firm, takes inside price ρ as given. Its profit-

[2]Note that consumers' surplus is absent from welfare as the good is assumed to be consumed abroad.

maximizing condition gives the reaction function of the domestic firm:

$$\rho(x) = \beta^p + 2\theta^p x. \tag{35}$$

In stage two, the foreign firm, who is the leader, maximizes profits π^f $(= (a - bx - \beta^p - 2\theta^p x - \beta^s - \theta^s x - t)x)$, taking into account the reaction function given in (??). The first-order profit-maximizing condition solves the output level and then inside price from (??) as:

$$\begin{aligned} x_{dp2} &= \frac{a - \beta^s - \beta^p - t}{2(b + \theta^s + 2\theta^p)}, \\ \rho_{dp2} &= \frac{(a - \beta^s + \beta^p - t)\theta^p + (b + \theta^s)\beta^p}{b + \theta^s + 2\theta^p}. \end{aligned}$$

The profit of the domestic firm and that of the foreign firm are calculated, respectively, as:

$$\pi^d_{p2} = \frac{(a - \beta^s - \beta^p - t)^2\theta^p}{4(b + \theta^s + 2\theta^p)^2}, \tag{36}$$

$$\pi^f_{s1} = \frac{(a - \beta^s - \beta^p - t)^2}{4(b + \theta^s + 2\theta^p)}. \tag{37}$$

As in the last subsection, in stage one, the domestic government chooses tax t to maximize the welfare of the home country as given in (??). In this case, the optimum tax is solved as:

$$t = \frac{(a - \beta^s - \beta^p)(b + \theta^s + \theta^p)}{(2b + 2\theta^s + 3\theta^p)} > 0. \tag{38}$$

As in the case when the domestic firm was a seller and for similar reasons, the optimal tax is higher when the domestic firm is the follower than when it is the leader.

By substituting (??) in (??) and (??), we get:

$$\begin{aligned} \pi^d_{p2} &= \frac{(a - \beta^s - \beta^p)^2\theta^p}{4(2b + 2\theta^s + 3\theta^p)^2}, \\ \pi^f_{s1} &= \frac{(a - \beta^s - \beta^p)^2(b + \theta^s + 2\theta^p)}{4(2b + 2\theta^s + 3\theta^p)^2}. \end{aligned}$$

3.2.3 Leadership preference

In this subsection, we shall check If there is a certain producer-seller relationship that both entities prefer, when the domestic firm is the producer. Comparing profits under the two scenarios, we find:

$$\pi^f_{s1} - \pi^f_{s2} = \frac{2(a-\beta^s-\beta^p)^2(\theta^p)^3}{4(2b+2\theta^s+3\theta^p)^2(2b+2\theta^s+\theta^p)^2} > 0,$$

$$\pi^d_{p1} - \pi^d_{p2} = \frac{(a-\beta^s-\beta^p)^2}{4(2b+2\theta^s+3\theta^p)^2(2b+2\theta^s+\theta^p)}\Delta^d > 0,$$

where

$$\Delta^d = (2b+2\theta^s+3\theta^p)^2 - (2b+2\theta^s+\theta^p)\theta^p > 0.$$

That is, in the absence of any side payments, both firms would prefer their own leadership. Although the optimal tax rate is lower when the domestic producer is the leader than when it is the follower, the difference in the optimal tax rates is bigger when the domestic firm is a seller. That is why, in this case, the foreign firm would never accept leadership by the domestic firm, but it would do under certain conditions (see proposition **??**) when the domestic firm is the seller, and the foreign firm is the producer.

Proposition 5 *In the absence of side payment, both firms would prefer their own leadership.*

In the presence of side payments, we consider again aggregate profits. The levels of, and the differences in, aggregate profits are:

$$\pi_{dp1} = \pi^d_{s2} + \pi^f_{p1} = \frac{(a-\beta^s-\beta^p)^2(3b+3\theta^s+\theta^p)}{4(2b+2\theta^s+\theta^p)^2},$$

$$\pi_{dp2} = \pi^d_{p2} + \pi^f_{s1} = \frac{(a-\beta^s-\beta^p)^2(b+\theta^s+3\theta^p)}{4(2b+2\theta^s+3\theta^p)^2},$$

$$\pi_{dp1} - \pi_{dp2} = \frac{(a-\beta^s-\beta^p)^2}{4(2b+2\theta^s+\theta^p)^2(2b+2\theta^s+3\theta^p)^2}\tilde{\Delta} > 0$$

Where

$$\tilde{\Delta} = (3b+3\theta^s+\theta^p)(2b+2\theta^s+3\theta^p)^2 - (2b+2\theta^s+\theta^p)^2(b+\theta^s+3\theta^p) > 0.$$

That is, in the presence of side payments, the foreign seller would to the leadership by the domestic producer. Formally,

Proposition 6 *In the presence of side payments, when domestic firm is the producer and the government imposes exports tax optimally, both firms would prefer domestic firms' leadership.*

Since the optimal tax is higher when the foreign firm is the leader, the foreign firm benefits sufficiently when it is not the leader and this over-compensates for the loss the foreign seller incurs being a follower.

3.3 Conclusion

In economic analysis, it is normally assumed that there is no difference between a producer and a seller, i.e., the producer direct sells to the consumer. But, in reality, more often than not, a producer sells its product to a seller and who then in turn sells the product to consumers. This vertical relationship between a producer and a consumer has been considered in the literature and it opens up new issues and problems. This paper is about one such problem: who should be the leader in such a vertical relationship. We consider several cases depending on the nationality of the two parties, on whether side payments between the two firms are allied, and on whether the domestic government is passive or active in setting optimal sales tax.

In te case where the government is passive, we find that, in the absence of any side payment, both firms will prefer their own leadership. However, when side payments are allowed it is possible that both parties agrre on the leadership of one of the two firms. We also find that when the government sets sales tax optimally, they can agree on the leadership issue even in the absence of side payments. This is true when the domestic firm is the seller. When side payments are allowed and the government is active, the leadership by the domestic firm emerges as the equilibrium.

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