

**Optimal Contracting with Wealth-Constrained Operators
of Unknown Ability**

Chifeng Dai
Department of Economics
Southern Illinois University Carbondale
Carbondale, IL 62901

Abstract

This paper examines how a project owner optimally selects a project operator and motivates him to deliver an unobservable effort when potential operators are wealth-constrained. It shows that either a pooling or a separating contract can arise in equilibrium. In a separating contract, the more capable potential operator is either selected more often but awarded a smaller share of profit, or selected less often but awarded a larger share of profit.

JEL Classification Numbers: D440, D820, L140.

Key words: Optimal contract; Wealth constraint; Asymmetric information; Allocation inefficiency

1. Introduction

Optimal contracts in the presence of moral hazard and adverse selection have received considerable attention in the literature, but most studies ignore wealth constraints. In practice, economic parties often are wealth-constrained. For example, CEOs in large corporations usually have little wealth relative to the assets they control. Entrepreneurs typically lack the funds required to develop and market their own inventions, and so seek the financial support of venture capitalists. Therefore, it is important to examine optimal contracts in the presence of wealth constraints.

This paper examines how a project owner optimally selects a project operator and motivates him to deliver unobservable effort. Potential operators have limited wealth and private knowledge of their ability. The wealth constraints are shown to influence optimal contracts in two fundamental ways. First, wealth constraints prevent the project owner from receiving the full value of the project. Furthermore, profit sharing results in the equilibrium, which diminishes the effort supplied by the operator. Second, wealth constraints can prevent a high-ability potential operator from outbidding his low-ability counterpart. As a result, the project sometimes is assigned to an operator of lower ability. Together, these two effects imply that wealth constraints give rise to diluted incentives and ex post allocation inefficiency in the equilibrium.

When the abilities of potential operators are common knowledge, the project owner always selects the more capable potential operator. However, more interestingly, the operator's share of realized profit can either increase or decrease with his ability, depending on the nature of his production technology. This is because shares of the realized profit constitute the only source of compensation for both the project owner and

the project operator when potential operators have no initial wealth. Under some production technologies, the project owner finds it optimal to motivate a more capable operator by awarding him a larger share of profit, because doing so induces a larger increase in the probability of success and therefore a larger increase in expected total surplus. However, under some other technologies, the project owner finds it optimal to award a more capable operator a smaller share of profit because doing so does not reduce the operator's effort substantially but secures a larger share of realized profit for the owner.

When the project owner cannot observe the abilities of the potential operators, she must consider both how to select the desired operator and how to motivate the selected operator. The project owner would prefer to select a high-ability potential operator more frequently, but the only way the project owner can do so without inducing the low-ability operator to exaggerate his ability is to couple a higher probability of operation with a smaller share of realized profit. However, a smaller share of profit reduces a more capable operator's incentive to deliver effort and therefore the expected total surplus. Under some production technologies, the project owner finds it is optimal to select a more capable operator more often but award him a smaller share of profit. Under other technologies, the effort reduction of a more capable operator resulting from a smaller share of profit becomes so large that the project owner finds a pooling contract optimal or even selects a more capable operator less often but awards him a larger share of profit (to ensure substantial effort supply). Consequently, when potential operators are privately informed about their abilities, either a pooling or a separating contract can arise in equilibrium. The nature of the optimal contract depends on the elasticity of an operator's

expected profit of operation with respect to the reward for success. In a separating contract, the more capable potential operator is either selected more often but awarded a smaller share of profit, or selected less often but awarded a larger share of profit.

This analysis extends the work of Lewis and Sappington (LS) [7, 8] who analyze a special case of the model considered here. LS adopt a functional form in which the key elasticity does not vary with the operator's ability. Consequently, for the reasons explained below, a separating equilibrium is optimal in the setting analyzed by LS, and the more capable operator is always selected more often but awarded a smaller share of profit. However, wealth constraints commonly exist in a broad class of economic settings that apparently have different production technologies. Also, both pooling contracts and separating contracts are frequently observed in many relevant practical settings. This study demonstrates that pooling contracts and other forms of separating contracts can be optimal in more general settings. Furthermore, it characterizes the optimal separating and pooling contracts for a broad class of production functions.

These findings are developed and explained as follows. Section II describes the central elements of our model. Section III examines the general properties of an optimal contract. Section IV summarizes the results and concludes with future research directions. The proofs of all formal results are provided in the appendix.

2. Elements of the Model

The owner of a project seeks to select an operator and motivate him to operate the project. The project will either succeed and provide a gross return $V > 0$, or fail and provide zero gross return. The success or failure of the project is observed publicly.

$p(\theta, e)$ is the probability that the project succeeds for $e \geq 0$, where e represents the effort that the operator delivers and θ represents the operator's ability. The operator's effort is not observable to the project owner. Effort is necessary for success so that $p(\theta, 0) = 0$. I assume that higher effort and ability increase the probability of success at a decreasing rate so that $p_\theta(\theta, e) > 0$, $p_e(\theta, e) > 0$, $p_{\theta\theta}(\theta, e) < 0$, $p_{ee}(\theta, e) < 0$. Higher ability is also assumed to increase the marginal impact of effort so that $p_{\theta e}(\theta, e) > 0$. In addition, marginal effect of effort at $e = 0$ is assumed to be sufficient large so that $\lim_{e \rightarrow 0^+} p_e(\theta, e) \rightarrow \infty$.

There are two potential operators in our model. Each potential operator's ability θ is the realization of an identical and independent random variable. The realization is θ_H with probability q and θ_L with probability $1 - q$, where $0 < \theta_L < \theta_H$. Only the potential operator is privately informed about his own ability from the outset. For simplicity, I assume that both potential operators have zero initial wealth and an opportunity wage of zero. Furthermore, I assume the potential operators have the same marginal cost of effort, which is constant and normalized to unity.

Let μ_{ij} denote the probability the owner assigns the project to a potential operator when he reports ability θ_i and his counterpart reports ability θ_j , for $i, j \in \{L, H\}$. Also let T_i denote the equilibrium payment the owner makes to the operator who reports ability θ_i when he is selected and the project succeeds. No transfer payment occurs between the owner and the operator when the project fails. Call $\{\mu_{ij}, T_i\}$ the allocation

that a potential operator receives when he reports ability θ_i and his counterpart reports θ_j , for $i, j \in \{L, H\}$.¹

The timing in the model is as follows: 1) The project owner offers the contract to both potential operators. 2) Each potential operator decides whether to accept the contract. No penalty is imposed on a potential operator who rejects the contract. 3) If both potential operators decide to accept the contract, each of them reports his own ability to the project owner.² 4) The owner selects a project operator according to the contract; 5) The selected operator chooses his effort level and manages the project; 6) At the end of the operation, the outcome of the project is observed, and payment is made as promised in the contract.

3. Properties of the Optimal Contract

3.2. A benchmark solution.

As a benchmark, I first examine the optimal contract when potential operators only have private information about their effort supply but not their abilities.

A selected operator chooses his effort level to maximize his profit of operating the project, which is the difference between the expected payment from the owner and the cost of his effort. Therefore, when T is the payment for success, an operator with ability θ chooses an effort level such that:

$$e(\theta, T) \equiv \underset{e}{\operatorname{argmax}} \{p(\theta, e)T - e\} \quad (2.1)$$

The equilibrium expected profit for a potential operator with ability θ_H is:

$$\pi(\theta_H, T) = p(\theta_H, e(\theta_H, T))T - e(\theta_H, T). \quad (2.2)$$

The corresponding equilibrium expected profit for a potential operator with ability θ_L is:

$$\pi(\theta_L, T) = p(\theta_L, e(\theta_L, T))T - e(\theta_L, T). \quad (2.3)$$

For each type of operator, the project owner wants to maximize her net expected return, which is the difference between the expected gross return from the project and the expected payment to the operator. Therefore, for each type of operator, the project owner's problem is:

$$\text{Max}_{T_i} W = p(\theta_i, e(\theta_i, T_i))[V - T_i]$$

Subject to

$$\pi(\theta_i, T_i) = p(\theta_i, e(\theta_i, T_i))T_i - e(\theta_i, T_i) \geq 0, \quad (2.4)$$

$$T_i \geq 0, \text{ where } i \in \{L, H\}. \quad (2.5)$$

Constraint (2.4) is the individual rationality constraint which guarantees the participation of both types of potential operators. Constraint (2.5) ensures the transfer payments to be nonnegative, which reflects the fact that both potential operators have zero initial wealth.

If the potential operators were not wealth-constrained, the project owner could charge the operator an up-front fee which equals the project's maximum expected value, $p(\theta, e^*(\theta))V - e^*(\theta)$, where $e^*(\theta) \equiv \operatorname{argmax}_e \{p(\theta, e)V - e\}$, and set $T = V$. Then the selected operator would choose to deliver the socially optimal effort $e^*(\theta)$ and he would earn zero expected profit. This mechanism is equivalent to selling the project to the operator at its maximum expected value. When both potential operators have zero initial wealth, the project owner's only source of compensation is her share in the

project. However, the share in the project is also the only source of incentive for the operator. This changes the properties of the optimal contract as shown in Proposition 1.

Proposition 1. Suppose each potential operator's ability is observable and wealth constraints exist. Then, in the optimal contract:

- (i) The project owner always shares the realized profit with the operator ($0 < T / V < 1$) who then delivers less than the socially optimal effort;
- (ii) The project owner's expected surplus increases as the selected operator's ability increases;
- (iii) The optimal payment to the operator for success can either increase or decrease with the operator's ability conditional on his production technology.

Because the share of the realized profit is the only source of incentive for the operator, as property (i) reports, the project owner has to promise to share the realized profit with the operator in order to motivate the operator. Since the project owner's only source of compensation is her share of the realized profit, the selected operator is rewarded only part of the realized profit. Consequently, he delivers less than the socially optimal level of effort $e^*(\theta)$. Given the optimal sharing of the realized profit, property (ii) indicates that the project owner's expected surplus increases as the operator's ability increases. Therefore, the project owner will always award the project to the potential operator with the highest ability when she can observe each potential operator's ability.

Furthermore, when the share of the project is the only source of compensation for both the project operator and the project owner, the project owner utilizes the high-ability operator's superior productivity in an interesting manner. The high-ability operator has a relatively high marginal probability of success at any given level of effort.

Thus, the project owner may prefer awarding a larger share of profit to a high-ability operator, because it can lead to a larger increase in the probability of success and therefore a larger increase in total surplus. On the other hand, the project owner may prefer awarding a smaller share of profit to a high-ability operator, because doing so may not reduce the operator's effort substantially but increases the share of realized profit that the owner keeps. Property (iii) of Proposition 1 shows that, in an optimal contract, the operator's share of profit can either increase or decrease with his ability, depending on the production technology. In other words, under some technologies, the operator's ability and the share of profit he receives act as substitutes from the owner's perspective. Under other technologies, they can act as complements.

When the operator's probability of success is $p(\theta, e) = \alpha e^\theta$, for example, the optimal share of profit for the operator is θ , which increases with θ . On the other hand, when the operator's probability of success is $p(\theta, e) = \theta e^\gamma$ as in LS, the optimal share of profit for the operator is γ , which does not vary with θ .

Proposition 2 provides a sufficient condition under which the owner will optimally award a high-ability operator a smaller share of profit.

Proposition 2: Suppose $p_{eee}(\theta, e) \leq 0$ and $p_{ee\theta}(\theta, e) \leq 0$, then when the abilities of operators are observed publicly, the owner will optimally award a larger share of profit to a low-ability operator than to a high-ability one.

While the conditions in Proposition 2 involve the third derivatives of the success function that are not easy to interpret, they are straightforward to check for any specific

function and they do indicate that it can be optimal to award a smaller share of the realized profit to a high-ability operator than to a low-ability operator.

For later use, define the class of production technologies, under which the operator's share of the realized profit decreases with his ability, the share-reversal production technology. Formally, let T^* be the optimal payment for success for an operator when his ability, θ , is observable to the project owner, then the operator's production technology is share-reversal if $\frac{dT^*}{d\theta} < 0$.

3.2. The optimal contract.

The results from the benchmark problem illustrate the effect of wealth constraints on the optimal contract when there is no adverse selection problem. However, in reality, a project owner often does not have perfect knowledge of potential operators' abilities. For example, venture capitalists seldom have perfect information about entrepreneurs' abilities, and company owners often are not able to assess perfectly the qualifications of potential managers. This section investigates the properties of optimal contracts when potential operators are privately informed about their abilities.

In this case, the equilibrium expected profit for a potential operator is the product of his expected profit from operation and his probability of operation. The equilibrium expected profit for a potential operator with ability θ_H is:

$$\Pi(\theta_H) = [p(\theta_H, e(\theta_H, T_H))T_H - e(\theta_H, T_H)] [q\mu_{HH} + (1-q)\mu_{HL}]. \quad (2.6)$$

The corresponding equilibrium expected profit for a potential operator with ability θ_L is:

$$\Pi(\theta_L) = [p(\theta_L, e(\theta_L, T_L))T_L - e(\theta_L, T_L)] [(1-q)\mu_{LL} + q\mu_{LH}]. \quad (2.7)$$

The project owner wants to maximize her net expected return, which is the difference between the expected gross return from the project and the expected payment to the operator. Therefore, the project owner's problem is:

$$\begin{aligned} \underset{T, \mu}{Max} \Pi^P = & [p(\theta_H, e(\theta_H, T_H))(V - T_H)][2q^2\mu_{HH} + 2q(1-q)\mu_{HL}] \\ & + [p(\theta_L, e(\theta_L, T_L))(V - T_L)][2(1-q)^2\mu_{LL} + 2q(1-q)\mu_{LH}] \end{aligned}$$

Subject to

$$\Pi(\theta_H) \geq 0; \quad (2.8)$$

$$\Pi(\theta_L) \geq 0; \quad (2.9)$$

$$\Pi(\theta_H) \geq [p(\theta_H, e(\theta_H, T_L))T_L - e(\theta_H, T_L)][q\mu_{LH} + (1-q)\mu_{LL}]; \quad (2.10)$$

$$\Pi(\theta_L) \geq [p(\theta_L, e(\theta_L, T_H))T_H - e(\theta_L, T_H)][q\mu_{HH} + (1-q)\mu_{HL}]; \quad (2.11)$$

$$T_H, T_L \geq 0; \quad (2.12)$$

$$\mu_{HL} + \mu_{LH} \leq 1; \quad (2.13)$$

$$\mu_{HH} \leq 1/2; \quad (2.14)$$

$$\mu_{LL} \leq 1/2; \quad (2.15)$$

$$\mu_{HH}, \mu_{HL}, \mu_{LH}, \mu_{LL} \geq 0. \quad (2.16)$$

The first term in the owner's objective function is the owner's expected return when the operator has ability θ_H times the probability that the selected operator has ability θ_H , and the second term is the owner's expected return when the operator has ability θ_L times the probability that the selected operator has ability θ_L . Therefore, the sum of these two terms equals the owner's expected net return from the project. (2.8) and (2.9) are the individual rationality constraints that guarantee the participation of both

types of potential operators. (2.10) and (2.11) are the incentive compatibility constraints that ensure both types of potential operators truthfully report their abilities. (2.12) ensures the transfer payments to be nonnegative. (2.13), (2.14), (2.15) and (2.16) bound the operation probabilities between 0 and 1.

When the project owner cannot observe the abilities of the potential operators, she must consider both how to select the desired operator and how to motivate the selected operator. The project owner could always assign the same contract to both potential operators and specify a particular share in the project and a particular probability of operation that does not vary with ability. However, as is evident in the benchmark problem, the project owner prefers to select a high-ability potential operator more frequently, because his greater productivity generates greater expected surplus for the project owner. The only way the project owner can do so without inducing the low-ability operator to exaggerate his ability is to couple a higher probability of operation with a smaller share of realized profit. The properties of optimal contracts with unobservable ability depend on whether the potential operators' production technology is share-reversal.

If their production technology is share-reversal, the project owner prefers to offer two separate contracts: (A) a relatively high probability of operation coupled with a relatively small share in the project, intended for a high-ability potential operator, and (B) a relatively low probability of operation coupled with a relatively large share in the project, intended for a low-ability potential operator.

However, if the potential operators' production technology is not share-reversal, the above separating contract may not be optimal. This is because while selecting a high-

ability operator more often can increase the project owner's expected surplus, reducing a high-ability operator's share of profit can substantially reduce his effort supply and consequently reduce the owner's expected surplus. When the latter effect dominates the former, the project owner may find a pooling contract is optimal or even prefer to select a high-ability operator less often but award him a larger share of profit (to ensure substantial effort supply). Therefore, when the potential operators' production technology is not share-reversal, the project owner's preference regarding separating contracts is not clear.

For example, T^* does not vary with the operator's ability θ when the operator's success probability is $p(\theta, e) = \theta e^\gamma$. In this case, a high-ability operator is always selected more often but awarded a smaller share of profit.³ On the other hand, T^* increases with the operator's ability θ when the operator's success probability is $p(\theta, e) = \alpha e^\theta$. Numerical examples in Table 1 shows that either a pooling contract or a separating contract can arise in equilibrium. Further, in a separating contract, a high-ability operator is selected less often but awarded a larger share of profit.

These findings are summarized in Proposition 3.

Proposition 3. Suppose potential operators are privately informed about their abilities from the outset and wealth constraints exist.

- (i) Either a pooling or separating contract can arise in equilibrium;
- (ii) The project owner always shares the realized profit with the operator, and the selected operator always earns positive expected profit;

- (iii) If the potential operators' production technology is share-reversal, the high-ability operator is always selected more often, but is awarded a smaller share of profit in a separating equilibrium;
- (iv) If the production technology is not share-reversal, the high-ability operator can be selected less often and awarded a larger share of profit in a separating equilibrium.

It remains to determine when the project owner prefers a separating contract and when he prefers a pooling contract. To analyze this issue, define $E_{\pi, T} \equiv \frac{d\pi(\theta, T)}{dT} \frac{T}{\pi(\theta, T)}$. In words, $E_{\pi, T}$ is the elasticity of a potential operator's expected profit of operating the project with respect to the payment for success.

Proposition 4. Suppose the potential operators' production technology is share-reversal. Then

- (i) A pooling contract is optimal when $E_{\pi, T}$ is strictly increasing in the operator's ability θ ;
- (ii) A separating contract is optimal when $E_{\pi, T}$ is non-increasing in the operator's ability θ .

The intuition underlying Proposition 4 is as follows. When the production technology is share reversal, the project owner prefers to select the high-ability operator more often but award him a smaller share of profit. However, when the elasticity of an operator's expected profit of operating the project with respect to the payment for success increases with his ability, the marginal rate of substitution between his probability of

operation and his payment of success, $MRS_{pT}^{\Pi} = \frac{\partial \Pi(\theta)/\partial T}{\partial \Pi(\theta)/\partial P}$, is also increasing with his ability. Consequently, a high-ability operator will be less willing than a low-ability operator to reduce his share of profit in exchange for a higher probability of operation. Therefore, as shown in Figure 1, given the two separate contracts A and B that we discussed earlier, a high-ability operator will prefer the option A designed for the low-ability operator in which a relatively low probability of operation is coupled with a relatively large share of profit. On the other hand, a low-ability operator will prefer the option B that couples a relatively high probability of operation with a relatively small share of profit. As a result, the project owner cannot implement any separating contracts with properties described in property (iii) of Proposition 3. In this case, a pooling contract is optimal. A pooling contract specifies the same share in the project and the same probability of operation (0.5) for all potential operators.

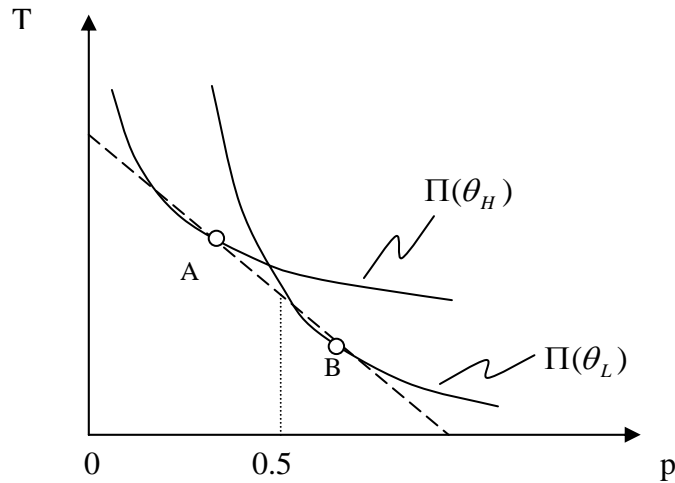


Figure 1. The Potential Operators' Preference for Contract Options When $E_{\pi, T}$ is Strictly Increasing in θ .

In contrast, when the aforementioned elasticity is non-increasing in the operator's ability, a high-ability operator is at least as willing to reduce his share of profit in exchange for a higher probability of operation as a low-ability operator is. Therefore, a high-ability operator will weakly prefer a relatively high probability of operation coupled with a relatively small share of profit, and a low-ability operator will weakly prefer a relatively low probability of operation coupled with a relatively large share of profit. As a result, separating contracts as characterized in property (iii) of Proposition 3 can be implemented, and are optimal.⁴

Proposition 5 provides two additional general properties regarding the optimal contract.

Proposition 5: Suppose potential operators are privately informed about their abilities from the outset and wealth constraints exist.

- (i) A separating contract, where a high-ability operator is selected more often but awarded a smaller share of profit, is not optimal if $E_{\pi, T}$ is strictly increasing in the operator's ability θ ;
- (ii) A separating contract, where a high-ability operator is selected less often but awarded a larger share of profit, is not optimal if $E_{\pi, T}$ is strictly decreasing in the operator's ability θ .

When the elasticity of an operator's expected profit of operating the project with respect to his payment for success is strictly increasing in his ability, a high-ability operator is less willing to reduce his share of profit than a low-ability operator in exchange for a higher probability of operation. Therefore, a separating contract, where a

high-ability operator is selected more often but awarded a smaller share of profit, cannot be implemented. In contrast, when the elasticity is strictly decreasing in ability, a high-ability operator is more willing to reduce his share of profit in exchange for a higher probability of operation. In this case, a separating contract, where a high-ability operator is selected less often but awarded a larger share of profit, cannot be implemented.

It can be verified that $E_{\pi, T}$ increases with the operator's ability θ when the operator's success probability is $p(\theta, e) = \alpha e^\theta$. The numerical examples in Table 1 reveal that the high-ability operator is selected less often but awarded a larger share of profit in separating contracts in this case, consistent with Proposition 5.

4. Conclusion

This study examines how a project owner optimally selects a project operator and motivates him to deliver unobservable effort when potential operators are wealth-constrained. It shows that wealth constraints have significant effects on the structure of optimal contracts. First, wealth constraints prevent the project owner from receiving the full value of the project and give rise to profit sharing in the equilibrium. Second, wealth constraints prevent a high-ability potential operator from outbidding his low-ability counterpart. As a result, the project can be assigned to potential operators of lower abilities. Consequently, diluted incentives and ex post allocation inefficiency arise in the equilibrium.

Further, it shows that when the abilities of potential operators are common knowledge, the operator's share of profit can either increase or decrease with his ability, depending on the prevailing production technology. When potential operators become

privately informed about their abilities, either a pooling or a separating contract can arise in the equilibrium⁵. In separating contracts, the more capable potential operator is either selected more often but awarded a smaller share of profit, or selected less often but awarded a larger share of profit. Which equilibrium arises depends upon the elasticity of operator's expected profit from operating the project with respect to the payment for success and whether the production technology is share reversal. It characterizes conditions for separating contracts to arise, and for pooling contracts to arise.

Our model could be usefully extended in a variety of directions. First, it could be optimal for the project owner to conduct preliminary contests among potential operators in order to better discern their abilities. The optimal design of such contests and the conditions under which such contests are optimal would be interesting to explore. Second, repeated interaction between the project owner and potential operators could be considered. Past performance can reveal information about an operator's ability. It can also create wealth asymmetries among potential operators. How the project owner optimally constructs future assignments and sharing rules based upon potential operators' past performance and heterogeneous wealth merits investigation.

FOOTNOTES

1. It is without loss of generality that the equilibrium payment is assumed a function of each potential operator's own report only. The reason is as follows. For any mechanism where each potential operator's payment does not depend only on his own report, there exists a corresponding payment scheme that provides potential operators the same expected profits but depends only on each potential operator's own report. Furthermore, since both project owner and potential operators are risk neutral and the production function is concave in effort, a deterministic payment scheme can render higher expected surplus to the project owner than a stochastic mechanism.
2. As shown in section 3, both types of potential operators make positive expected profit from the optimal contract. Therefore, in the equilibrium, both potential operators will accept the optimal contract regardless of their abilities.
3. See LS.
4. It can be verified that $E_{\pi, T}$ is constant in the operator's ability θ when the operator's success probability is $p(\theta, e) = \theta e^\gamma$ as in LS, which is consistent with Proposition 4.
5. There are special classes of production functions for which pooling is always induced in equilibrium. For example, suppose $p(\theta, e) = p(X)$, where $X = X(\theta, e)$ and $X_e(\theta, e) = C$ (C is a scalar.). Then the first order condition of the operator's problem implies $p_X(X)X_e(\theta, e)T = p_X(X)CT = 1$. Therefore, the X chosen by the operator

does not depend on his ability level. In the presence of information asymmetry and wealth constraints, the project owner is indifferent between different types of potential operators because she is not able to extract additional rent from the high-ability operator. A simple example of this class of production functions is

$$p(\theta, e) = 1 - \frac{1}{(\theta + e)^r}.$$

Table 1

Optimal Solutions for various values of θ_H , θ_L and q when $V=2000$, and $\alpha=0.001$.

$$(p(\theta, e) = \alpha e^\theta)$$

	θ_H	θ_L	q	T_H	T_L	μ_{HL}	μ_{LH}
1	0.80	0.80	0.70	1600.00	1600.00	0.50	0.50
2	0.80	0.75	0.70	1581.75	1581.75	0.50	0.50
3	0.80	0.70	0.70	1575.08	1575.08	0.50	0.50
4	0.80	0.65	0.70	1575.08	1575.08	0.50	0.50
5	0.80	0.60	0.70	1570.84	1570.84	0.50	0.50
6	0.80	0.55	0.70	1570.84	1570.84	0.50	0.50
7	0.80	0.50	0.70	1570.84	1570.84	0.50	0.50
8	0.80	0.45	0.70	1570.84	1570.84	0.50	0.50
9	0.80	0.40	0.70	1570.84	1570.84	0.50	0.50
10	0.80	0.35	0.70	1572.53	1290.00	0.34	0.66
11	0.80	0.30	0.70	1575.86	1063.25	0.19	0.81
12	0.80	0.25	0.70	1580.05	812.19	0.00	1.00
13	0.80	0.20	0.70	1575.77	774.84	0.00	1.00
14	0.80	0.15	0.70	1570.76	738.86	0.00	1.00
15	0.80	0.10	0.70	1564.64	704.04	0.00	1.00
16	0.70	0.70	0.35	1400.00	1400.00	0.50	0.50
17	0.70	0.65	0.35	1362.07	1326.02	0.46	0.54
18	0.70	0.60	0.35	1350.17	1238.77	0.40	0.60
19	0.70	0.55	0.35	1350.78	1139.89	0.32	0.68
20	0.70	0.50	0.35	1360.03	1032.46	0.25	0.75
21	0.70	0.45	0.35	1372.13	919.44	0.18	0.82
22	0.70	0.40	0.35	1385.71	803.53	0.12	0.88
23	0.70	0.35	0.35	1390.34	699.87	0.00	1.00
24	0.70	0.30	0.35	1393.24	599.95	0.00	1.00

25	0.70	0.25	0.35	1394.92	506.82	0.00	1.00
26	0.70	0.20	0.35	1316.33	447.04	0.00	1.00
27	0.70	0.15	0.35	1210.48	384.27	0.00	1.00
28	0.70	0.10	0.35	1032.22	306.29	0.00	1.00

APPENDIX

Proof of Proposition 1:

Proof of Property (i):

For $\theta \in \{ \theta_L, \theta_H \}$, $\lim_{e \rightarrow 0^+} p_e(\theta, e) \rightarrow \infty$ guarantee that the operator will make positive expected profit whenever $T > 0$. Furthermore, $\Pi^P > 0$ if $V > T > 0$, while $\Pi^P = 0$ if $T = V$, or $T = 0$. So, in equilibrium, we have $V > T > 0$, and both types of operators can make positive expected profit from operation.

Q.E.D

Proof of property (ii):

Taking the partial derivative of Π^P with respect to θ and applying the Envelope Theorem provides

$$\frac{\partial \Pi^P}{\partial \theta} = [p_\theta(\theta, e(\theta, T)) + p_e(\theta, e(\theta, T)) e_\theta(\theta, T)](V - T) > 0. \quad (\text{A.1})$$

Q.E.D

Proof of Proposition 2:

The first order condition of the project owner's problem is:

$$\frac{\partial \Pi^P}{\partial T} = -p(\theta, e(\theta, T)) + (V - T)p_e(\theta, e(\theta, T)) e_T(\theta, T) = 0. \quad (\text{A.2})$$

From the operator's problem,

$$p_e(\theta, e(\theta, T)) = 1/T. \quad (\text{A.3})$$

$$\text{Then, } p_e(\theta, e(\theta, T))T = 1. \quad (\text{A.4})$$

Totally differentiating equation (A.4) and applying the Envelope Theorem provides

$$p_{ee}(\theta, e(\theta, T))Tde + p_e(\theta, e(\theta, T))dT + p_{e\theta}(\theta, e(\theta, T))Td\theta = 0. \quad (\text{A.5})$$

$$\text{So, } \frac{de}{dT} = -\frac{p_e(\theta, e(\theta, T))}{p_{ee}(\theta, e(\theta, T))T} = -\frac{1}{p_{ee}(\theta, e(\theta, T))T^2} > 0, \quad (\text{A.6})$$

$$\text{and } \frac{de}{d\theta} = -\frac{p_{e\theta}(\theta, e(\theta, T))}{p_{ee}(\theta, e(\theta, T))} > 0. \quad (\text{A.7})$$

Substituting (A.4) and (A.6) into (A.2) implies

$$\frac{\partial \Pi^P}{\partial T} = -p(\theta, e(\theta, T)) - (V - T) \frac{1}{p_{ee}(\theta, e(\theta, T))T^3} = 0. \quad (\text{A.8})$$

Total differentiating equation (A.8) gives

$$\frac{dT}{d\theta} = \frac{-p_e(\theta, e) e_\theta(\theta, T) - p_\theta(\theta, e) + \left[\frac{V - T}{T^3} \right] \left[\frac{p_{eee}(\theta, e) e_\theta(\theta, T) + p_{ee\theta}(\theta, e)}{p_{ee}(\theta, e(\theta, T))^2} \right]}{p_e(\theta, e) e_T(\theta, T) - \left\{ \frac{1}{p_{ee}(\theta, e)T^3} + (V - T) \left[\frac{3}{p_{ee}(\theta, e)T^4} + \frac{p_{eee}(\theta, e) e_T(\theta, T)}{p_{ee}(\theta, e)^2 T^3} \right] \right\}}. \quad (\text{A.9})$$

Since $p_\theta(\theta, e)$, $p_e(\theta, e) > 0$, $p_{ee}(\theta, e) < 0$, $p_{eee}(\theta, e) \leq 0$ and

$$p_{ee\theta}(\theta, e) \leq 0, \text{ equation (A.9) implies } \frac{dT}{d\theta} < 0.$$

Taking the derivative of (A.8) with respect to T again provides

$$\frac{\partial^2 \Pi^P}{\partial T^2} = -p_e(\theta, e) e_T(\theta, T) + \left[\frac{3V - 2T}{p_{ee}(\theta, e)T^4} + \frac{(V - T)p_{eee}(\theta, e) e_T(\theta, T)}{p_{ee}(\theta, e)^2 T^3} \right] < 0, \quad (\text{A.10})$$

which implies that the second-order condition with respect to T is satisfied.

Q.E.D

Lemma 1: In the optimal contract, $\mu_{HL} + \mu_{LH} = 1$.

Proof: Assume that $\mu_{HL} + \mu_{LH} + \varepsilon = 1$ and $0 < \varepsilon < 1$ in the optimal contract.

Furthermore, define

$$\tau_1 = [q\mu_{HH} + (1-q)\mu_{HL}]/(1-q)\sigma \text{ and } \tau_2 = [q\mu_{LH} + (1-q)\mu_{LL}]/q\sigma .$$

σ is a scalar which satisfies $\sigma \geq \frac{q\mu_{HH} + (1-q)\mu_{HL}}{(1-q)\varepsilon} + \frac{q\mu_{LH} + (1-q)\mu_{LL}}{q\varepsilon}$ so that

$$0 < \tau_1 + \tau_2 \leq \varepsilon .$$

Now define $\mu_{HL}^\circ = \mu_{HL} + \tau_1$ and $\mu_{LH}^\circ = \mu_{LH} + \tau_2$ and replace μ_{HL} and μ_{LH} in the presumed optimal contract with μ_{HL}° and μ_{LH}° , respectively.

Constraint (2.10) becomes

$$(1 + \frac{1}{\sigma})\Pi(\theta_H) \geq (1 + \frac{1}{\sigma})[p(\theta_H, e(\theta_H, T_L))T_L - e(\theta_H, T_L)][q\mu_{LH} + (1-q)\mu_{LL}] , \quad (\text{A.11})$$

and constraint (2.11) becomes

$$(1 + \frac{1}{\sigma})\Pi(\theta_L) \geq (1 + \frac{1}{\sigma})[p(\theta_L, e(\theta_L, T_H))T_H - e(\theta_L, T_H)][q\mu_{HH} + (1-q)\mu_{HL}] . \quad (\text{A.12})$$

(A.11) and (A.12) imply both constraints (2.10) and (2.11) still hold. And it is apparent that all the other constraints still hold while the value of Π^P has increased.

Therefore $\mu_{HL} + \mu_{LH} + \varepsilon = 1$ can not be part of optimal contract.

Q.E.D.

Lemma 2: In the optimal contract, $\mu_{HH} = \mu_{LL} = 1/2$.

Proof: Assume $\mu_{HH} + \varepsilon = 1/2$ and $0 < \varepsilon < 1/2$ in the optimal contract, and define

$\mu_{HH}^\circ = \mu_{HH} + \varepsilon = 1/2$ and $\mu_{HL}^\circ = \mu_{HL} - \frac{1-q}{q}\varepsilon$. Now replace μ_{HH} and μ_{HL} in the

presumed optimal contract with μ_{HH}° and μ_{HL}° . Since $q\mu_{HH} + (1-q)\mu_{HL}$ is the only term in constraints (2.10) and (2.11) that has μ_{HH} and μ_{HL} , and

$$\begin{aligned} q\mu_{HH}^\circ + (1-q)\mu_{HL}^\circ &= \frac{1}{2}q + (1-q)\left(\mu_{HL} - \frac{q}{1-q}\varepsilon\right) = q\left(\frac{1}{2} - \varepsilon\right) + (1-q)\mu_{HL} \\ &= q\mu_{HH} + (1-q)\mu_{HL}. \end{aligned} \tag{A.13}$$

(A.13) implies both constraints (2.10) and (2.11) still hold. And it is apparent that all the other constraints holds. Then we check the value of Π^P . Since $2q^2\mu_{HH} + 2q(1-q)\mu_{HL}$ is the only term in Π^P that has μ_{HH} and μ_{HL} ,

$$\begin{aligned} 2q^2\mu_{HH}^\circ + 2q(1-q)\mu_{HL}^\circ &= 2q^2(\mu_{HH} + \varepsilon) + 2q(1-q)\left(\mu_{HL} - \frac{q}{1-q}\varepsilon\right) \\ &= 2q^2\mu_{HH} + 2q(1-q)\mu_{HL}, \end{aligned} \tag{A.14}$$

(A.14) implies the value of Π^P are still the same. Therefore replacing μ_{HH} and μ_{HL} in the presumed optimal contract with μ_{HH}° and μ_{HL}° gives us an contract which is equivalent to the original contract.

Since $\mu_{HL} + \mu_{LH} \leq 1$ and $\mu_{HL}^\circ = \mu_{HL} - \frac{1-q}{q}\varepsilon$, we know $\mu_{HL}^\circ + \mu_{LH} < 1$, which is

a contradiction to Lemma 1. Therefore $\mu_{HH} < 1/2$ can not be part of optimal contract.

Applying the same argument to μ_{LL} , we can show that $\mu_{LL} = 1/2$ in the optimal contract.

Q.E.D

Lemma 3: In equilibrium, the project owner receives higher expected surplus when a more capable operator is selected.

First define $\bar{\Pi}^P$ as the project owner's expected net return from the optimal pooling contract, and T_H^* and T_L^* as the optimal payments for potential operators when their abilities are publicly observed, where the subscript represents the operator's ability level. For convenience, let

$$\pi_H^P = p(\theta_H, e(\theta_H, T_H))(V - T_H) \text{ and } \pi_L^P = p(\theta_L, e(\theta_L, T_L))(V - T_L).$$

Then project owner's objective function is

$$\Pi^P = \pi_H^P [2q^2 \mu_{HH} + 2q(1-q)\mu_{HL}] + \pi_L^P [2(1-q)^2 \mu_{LL} + 2q(1-q)\mu_{LH}]. \quad (\text{A.15})$$

(A.15) and Lemma 2 imply

$$\Pi^P = \pi_H^P [q^2 + 2q(1-q)\mu_{HL}] + \pi_L^P [(1-q)^2 + 2q(1-q)\mu_{LH}]. \quad (\text{A.16})$$

Suppose $\pi_H^P \leq \pi_L^P$ in the optimal contract. Then (A.15) implies

$$\Pi^P \leq \pi_L^P [q^2 + 2q(1-q)\mu_{HL}] + \pi_L^P [(1-q)^2 + 2q(1-q)\mu_{LH}] = \pi_L^P. \quad (\text{A.17})$$

Since $p(\theta_L, e(\theta_L, T_L^*))(V - T_L^*)$ is the owner's expected net return from a low-ability operator's operation when she can observe potential operators' ability levels,

$$\pi_L^P \leq p(\theta_L, e(\theta_L, T_L^*))(V - T_L^*). \quad (\text{A.18})$$

And it is easy to show that

$$p(\theta_H, e(\theta_H, T_L^*))(V - T_L^*) > p(\theta_L, e(\theta_L, T_L^*))(V - T_L^*). \quad (\text{A.19})$$

(A.17), (A.18) and (A.19) imply

$$\Pi^P < p(\theta_H, e(\theta_H, T_L^*))(V - T_L^*)q + p(\theta_L, e(\theta_L, T_L^*))(V - T_L^*)(1-q). \quad (\text{A.20})$$

By definition,

$$\bar{\Pi}^P \geq p(\theta_H, e(\theta_H, T_L^*)) (V - T_L^*) q + p(\theta_L, e(\theta_L, T_L^*)) (V - T_L^*) (1 - q). \quad (\text{A.21})$$

Therefore $\Pi^P < \bar{\Pi}^P$, which implies the presumed solution is not an optimal solution.

Q.E.D.

Proof of Proposition 3:

Proof of Property (iii):

Suppose $T_H > T_L$ in the equilibrium. Then constraint (2.11) implies that

$$q\mu_{LH} + (1 - q)\mu_{LL} > q\mu_{HH} + (1 - q)\mu_{HL}. \quad (\text{A.22})$$

From Lemma 1 and 2, we know $\mu_{HH} = \mu_{LL} = 1/2$ and $\mu_{HL} = 1 - \mu_{LH}$. Then (A.22)

implies

$$\mu_{HL} < 1/2 \text{ and } \mu_{LH} > 1/2. \quad (\text{A.23})$$

Also note that Property (ii) in Proposition 1 implies $T_H^* < T_L^*$ and (A.10) implies

$$\frac{\partial^2 [p(\theta, e(\theta, T))(V - T)]}{\partial T^2} < 0. \quad (\text{A.24})$$

Three cases need to be checked.

Case I: $T_H > T_L \geq T_L^*$;

Lemma 4 implies that $\pi_H^P > \pi_L^P$. Therefore (A.16) and (A.23) imply

$$\Pi^P < \pi_H^P [q^2 + 2q(1 - q) \frac{1}{2}] + \pi_L^P [(1 - q)^2 + 2q(1 - q) \frac{1}{2}] = \pi_H^P q + \pi_L^P (1 - q) \quad (\text{A.25})$$

Since $T_H > T_L \geq T_L^* > T_H^*$, (A.18) and (A.24) imply

$$\pi_H^P q + \pi_L^P (1 - q) < p(\theta_H, e(\theta_H, T_L^*)) (V - T_L^*) q + p(\theta_L, e(\theta_L, T_L^*)) (V - T_L^*) (1 - q)$$

$$< \bar{\Pi}^P . \quad (\text{A.26})$$

Therefore $\Pi^P < \bar{\Pi}^P$, which implies the presumed solution is not an optimal solution.

Case II: $T_H^* \geq T_H > T_L$;

From (A.25), $\Pi^P < \pi_H^P q + \pi_L^P (1 - q)$. Since $T_L^* > T_H^* \geq T_H > T_L$, and by definition $\pi_H^P \leq p(\theta_H, e(\theta_H, T_H^*))(V - T_H^*)$, (A.24) implies

$$\begin{aligned} \pi_H^P q + \pi_L^P (1 - q) &< p(\theta_H, e(\theta_H, T_H^*))(V - T_H^*)q + p(\theta_L, e(\theta_L, T_H^*))(V - T_H^*)(1 - q) \\ &< \bar{\Pi}^P . \end{aligned} \quad (\text{A.27})$$

Therefore $\Pi^P < \bar{\Pi}^P$, which implies the presumed solution is not an optimal solution.

Case III: $T_L^* \geq T_H > T_L \geq T_H^*$.

Again, from (A.25), $\Pi^P < \pi_H^P q + \pi_L^P (1 - q)$.

Since $T_L^* \geq T_H > T_L \geq T_H^*$, (A.24) implies

$$\begin{aligned} \pi_H^P q + \pi_L^P (1 - q) &< p(\theta_H, e(\theta_H, \frac{T_H^* + T_L^*}{2}))(V - \frac{T_H^* + T_L^*}{2})q \\ &\quad + p(\theta_L, e(\theta_L, \frac{T_H^* + T_L^*}{2}))(V - \frac{T_H^* + T_L^*}{2})q \leq \bar{\Pi}^P . \end{aligned} \quad (\text{A.28})$$

Therefore $\Pi^P < \bar{\Pi}^P$, which implies the presumed solution is not an optimal solution.

Q.E.D.

Proof of Propostion 4:

Proof of Property (i):

Constraint (2.10) requires that

$$\frac{\pi(\theta_H, T_H)}{\pi(\theta_H, T_L)} \geq \frac{q\mu_{LH} + (1-q)\mu_{LL}}{q\mu_{HH} + (1-q)\mu_{HL}}, \quad (\text{A.29})$$

and constraint (2.11) requires

$$\frac{q\mu_{LH} + (1-q)\mu_{LL}}{q\mu_{HH} + (1-q)\mu_{HL}} \geq \frac{\pi(\theta_L, T_H)}{\pi(\theta_L, T_L)}. \quad (\text{A.30})$$

Therefore, constraints (2.10) and (2.11) together requires

$$\frac{\pi(\theta_H, T_H)}{\pi(\theta_H, T_L)} \geq \frac{\pi(\theta_L, T_H)}{\pi(\theta_L, T_L)}. \quad (\text{A.31})$$

It can be shown that $\frac{d}{dT} \left(\frac{\pi(\theta_H)}{\pi(\theta_L)} \right) > 0$, when $\frac{dE_{\pi, T}}{d\theta} > 0$. Since $T_H < T_L$ in any

separating contracts according to Proposition 2, Condition (A.31) cannot hold when

$\frac{d}{dT} \left(\frac{\pi(\theta_H)}{\pi(\theta_L)} \right) > 0$. Therefore, no separating contract is optimal when $\frac{dE_{\pi, T}}{d\theta} > 0$.

Q.E.D

Proof of Property (ii):

Assume that a pooling contract is optimal. Define \bar{T} as the optimal payment for success in the pooling contract. Apparently each potential operator's probability of operating is 1/2.

According to Proposition 2, it is straightforward that $T_L^* > \bar{T} > T_H^*$, where T_H^* and T_L^* are the optimal payment of success for corresponding potential operator when the project owner can observe the potential operator's ability.

Define $T_H \equiv \bar{T} - \Delta T$, where $\bar{T} - T_H^* \geq \Delta T > 0$. We further define $\Delta\mu_{HL}$ so that

$$\left[p(\theta_H, e(\theta_H, T_H))T_H - e(\theta_H, T_H) \right] \left[\frac{1}{2} + (1-q)\Delta\mu_{HL} \right] \equiv \left[p(\theta_H, e(\theta_H, \bar{T}))\bar{T} - e(\theta_H, \bar{T}) \right] \left[\frac{1}{2} - q\Delta\mu_{HL} \right]. \quad (\text{A.32})$$

By continuity,

$$\frac{1}{2} < \frac{1}{2} + \Delta\mu_{HL} < 1 \quad (\text{A.33})$$

as ΔT is sufficiently close to zero.

From (A.32),

$$\frac{\frac{1}{2} + (1-q)\Delta\mu_{HL}}{\frac{1}{2} - q\Delta\mu_{HL}} \equiv \frac{p(\theta_H, e(\theta_H, \bar{T}))\bar{T} - e(\theta_H, \bar{T})}{p(\theta_H, e(\theta_H, \bar{T} - \Delta T))(\bar{T} - \Delta T) - e(\theta_H, \bar{T} - \Delta T)}. \quad (\text{A.34})$$

Since $\frac{d}{dT} \left(\frac{\pi(\theta_L)}{\pi(\theta_H)} \right) < 0$,

$$\frac{p(\theta_H, e(\theta_H, \bar{T}))\bar{T} - e(\theta_H, \bar{T})}{p(\theta_H, e(\theta_H, \bar{T} - \Delta T))(\bar{T} - \Delta T) - e(\theta_H, \bar{T} - \Delta T)} > \frac{p(\theta_L, e(\theta_L, \bar{T}))\bar{T} - e(\theta_L, \bar{T})}{p(\theta_L, e(\theta_L, \bar{T} - \Delta T))(\bar{T} - \Delta T) - e(\theta_L, \bar{T} - \Delta T)}. \quad (\text{A.35})$$

Therefore,

$$\frac{\frac{1}{2} + (1-q)\Delta\mu_{HL}}{\frac{1}{2} - q\Delta\mu_{HL}} > \frac{p(\theta_L, e(\theta_L, \bar{T}))\bar{T} - e(\theta_L, \bar{T})}{p(\theta_L, e(\theta_L, \bar{T} - \Delta T))(\bar{T} - \Delta T) - e(\theta_L, \bar{T} - \Delta T)}. \quad (\text{A.36})$$

Furthermore, as $p(\theta_H, e(\theta_H, T_H))(V - T_H) > p(\theta_H, e(\theta_H, \bar{T}))(V - \bar{T})$ and $p(\theta_H, e(\theta_H, T_H))(V - T_H) > p(\theta_L, e(\theta_L, \bar{T}))(V - \bar{T})$, the project owner is strictly better off with a contract in which the payments of success for the high-ability potential operator and the low-ability operator respectively are T_H and \bar{T} , and the probabilities of operation are $\mu_{HH} = \mu_{LL} = \frac{1}{2}$, $\mu_{HL} = \frac{1}{2} + \Delta\mu_{HL}$ and $\mu_{LH} = \frac{1}{2} - \Delta\mu_{HL}$. Based upon (A.32), (A.33) and (A.36), It can be readily verified that this new contract satisfies all the constraints.

Therefore, a pooling contract is not optimal.

Q.E.D.

REFERENCES

- [1] Che, Y and Gale, I., Standard Auctions with Financially Constrained Bidders. *Review of Economic Studies* **65** (1998), 1-22.
- [2] Harris, H. and Townsend, R., Allocation Mechanism, Asymmetric Information, and the Revelation Principle. in G. Feiwel (ed.), *Issues in contemporary Microeconomics and Welfare* (London: MacMillan, 1985), 379-394.
- [3] Laffont, J. and Matoussi, S. M., Moral Hazard, Financial Constraints, and Sharecropping In El Oulja. *Review of Economic Studies* **62** (1995), 381-400.
- [4] Laffont, J. and Robert, J., Optimal Auction with Financially Constrained Buyers. *Economics Letters* **52** (1996), 181-186.
- [5] Laffont, J. and Tirole, J., Auction Incentive Contracts. *Journal of Political Economy* **95** (1987), 921-937.
- [6] Laffont, J. and Tirole, J., *A theory of incentives in procurement and regulation*, (Cambridge MA: M.I.T. Press, 1993).
- [7] Lewis, T. and Sappington, D., Motivating Wealth-Constrained Actors. *American Economic Review* **90** (2000a), 944-957.
- [8] Lewis, T. and Sappington, D., Contracting with Wealth-Constrained Agents. *International Economic Review* **41** (2000b), 743-767.
- [9] Riordan M. and Sappington, D., Awarding Monopoly Franchises. *American Economic Review* **77** (1987), 375-387.
- [10] Sappington, D., Limited Liability Contract between Principal and Agent. *Journal of Economic Theory* **29** (1983), 1-21.