Real Exchange Rate Dynamics: The Role of Elastic Labor Supply

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Abstract

Empirical evidence suggests that the flexibility of labor supply is closely related to the dynamic adjustment of the real exchange rate. This paper investigates this relationship in a two-sector dependent economy model. While, the long-run equilibrium real exchange rate is independent of the elasticity of labor supply, our analysis confirms that the nature of the labor supply can be a crucially important determinant of its short-run dynamics. The extent to which this is so depends to some degree on the source of the underlying structural change that is driving the dynamics of the real exchange rate. Numerical simulations confirm that this mechanism may help explain the larger short-run volatility and more rapid convergence typically associated with developing countries having less flexible labor markets.

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1. Introduction

The real exchange rate (RER) puzzle has spawned an extensive literature, stimulating researchers to propose different explanations [Rogoff, 1996; Betts and Devereux, 1996; Hau, 2000; Obstfeld and Rogoff, 2001; Bergin and Feenstra, 2001; Chari, Kehoe, and McGrattan, 2002; Devereux and Engel, 2002; Morshed and Turnovsky, 2004; Chen and Hsu, 2009; Carvalho and Nechio, 2010]. Two key aspects of the puzzle are: (i) the long-term persistence of the real exchange rate following a structural change, and (ii) its short-term volatility, both of which exhibit systematic patterns across economies. With respect to the persistence of the real exchange rate, the rate of convergence to its long-run equilibrium value is significantly slower for developed countries than it is for developing countries [Cheung and Lai, 2000]. These authors examined a number of structural characteristics such as inflation, productivity growth, trade openness, and the size of government spending to account for these cross-country differences in the rate of convergence. They observe that only inflation and government spending yield a weak relationship with the observed pattern of persistence. Consequently, their findings underscore the need to identify the determinants of the persistence of the PPP deviation in order to explain these differences.1 With regard to volatility, Hausmann, Panizza, and Rigobon (2006) show that developing countries have substantially more volatile real exchange rates than do developed countries, a difference that cannot be explained by differences in the magnitudes of the underlying shocks.2 In addition, Hau (2002) finds that increased openness is associated with less volatility in the real exchange rate.3

A natural framework for addressing the dynamics of the real exchange rate is the dynamic “dependent-economy model”, which determines the real exchange rate within a two-sector production framework.4 But if one employs the standard Heckscher-Ohlin production structure, in

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1 In a separate, but related, strand of literature, researchers found that price convergence in cities is faster for developing countries than in developed countries [Cecchetti et al., 2002; Chen and Devereux, 2003, Sonora, 2005; and Morshed et al., 2006].
2 Ganguly and Boucher (2010) show how the inclusion of various nominal factors can reduce the spread in volatility between developed and less developed economies.
3 Novy (2010) obtains a similar result, showing that trade costs, which reduce trade flows, thereby reducing openness, have the effect of increasing exchange rate volatility.
4 Several versions of the “dependent economy model”, emphasizing the difference between traded and nontraded goods can be identified. These include: the “Australian” model [e.g. Salter, 1959, Swan, 1960], the Balassa-Samuelson model [Balassa, 1964, Samuelson, 1964], the “Scandinavian” model [e.g. Aukrust, 1970, Lindbeck, 1979], and the contribution from Latin America due to Diaz-Alejandro (1965). These early contributions were static and focused on different
which the aggregate labor supply is fixed and the productive factors are perfectly mobile across sectors, the model is unable to generate plausible real exchange rate dynamics. Depending upon relative sectoral capital intensities, the real exchange rate adjusts too rapidly or even worse, instantaneously.\(^5\)

To generate realistic exchange rate dynamics some source of sluggishness must be introduced into the adjustment process. Steigum and Thørgesen (2003) and Morshed and Turnovsky (2004) do so by relaxing the conventional assumption that capital can be instantaneously and costlessly shipped across sectors. Instead, they assume that the intersectoral movement of capital involves adjustment costs, reflecting the costs of retrofitting, an idea that can be traced back to Mussa (1978) and later to Gavin (1990, 1992).\(^6\)

One key element of the adjustment process involves the role of differences in the production structures between rich and poor countries in explaining differences in the dynamics of the RER. An empirical study by Duffy and Papageorgiou (2000) suggests that the elasticity of substitution between capital and labor exceeds 1 in rich countries, but is less than 1 in poor countries. Drawing upon this empirical evidence, Morshed and Turnovsky (2006) show that the more rapid speed of convergence of the RER in a developing economy may be explained by its lower elasticity of substitution in production.\(^7\)

But production flexibility is in part constrained by labor supply, and in this regard most dependent economy models assume that aggregate labor is supplied inelastically. Thus, one potentially important dimension that has been virtually ignored in discussing RER dynamics is the

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5 See e.g. Turnovsky (1997).
6 Chen and Hsu (2009) apply the Morshed-Turnovsky model to the Blanchard (1985) finite horizon model. Craighead (2009) shows how the introduction of costly intersectoral labor adjustments increases the volatility of the real exchange rate. In contrast to these studies, which rely on impediments to sectoral factor mobility, most of the previous literature addresses the issue of RER dynamics by introducing sticky goods prices; see e.g. Obstfeld and Rogoff (1995), Betts and Devereux (1996, 2000), Bergin and Feenstra (2001), Chari et al. (2002), and Ng (2003). In addition, Hau (2000) introduces sticky factor price while Carvalho and Neehio (2010) introduce firms that are different in the extent of price stickiness. Devereux and Engel (2002) emphasize price formation with special emphasis on local currency pricing, while Rogoff (1996) and Obstfeld and Rogoff (2001) introduce market segmentation resulting from trade frictions.
7 Other possible explanations for the differences in the rates of convergence of the RER between rich and poor countries authors include a variety of issues like fiscal policy, trade policy, industrial policy, country differences in price formation, and different adjustment costs [Cheung and Lai, 2000, Hau, 2002, Alba and Papell, 2007].
role of the labor-leisure choice. We analyze the dynamics of the RER (defined as the price of nontraded goods in terms of traded goods as numeraire), by incorporating the labor-leisure choice into a dynamic two-sector dependent economy model, in which inter-sectoral capital movements involve adjustment costs. Endogenizing labor (leisure) requires us to focus on the time allocation of the representative agent between total labor supply and leisure, on the one hand, and the allocation of labor between the traded and nontraded sectors, on the other.

The key mechanism whereby labor supply influences the RER is through its response to changing wealth. As an agent accumulates wealth, its marginal utility declines, causing the agent to supply less labor. This reduces the marginal productivity of capital, and discourages capital accumulation, which in turn is reflected in the accumulation of foreign bonds. But it also increases the consumption of all goods, including leisure. Consequently, as an individual adjusts his labor supply in response to some structural or policy change, this is likely to alter the mix between traded and non-traded goods, depending on their relative sectoral capital intensities, and this, in turn, will have important consequences for real exchange rate dynamics. In the case where the structural change involves a productivity increase, which directly raises the real wage, this too will affect labor supply and will be an additional channel whereby the endogenous labor supply influences the RER.

To illustrate the importance of the labor-leisure choice on cross-country differences in the RER dynamics, we have compiled the rate of convergence of the RER in a number of countries from Murray and Papell (2005) for a number of OECD countries and average annual hours actually worked per person in employment data from OECD (shown in Table 1).\(^8\) The correlation coefficient of half-life and work hours is 0.40 and it is highly significant. This implies that the greater is the consumption of leisure, the faster is the rate of convergence (shorter half-life). This suggests that the adjustment of the labor supply is potentially a significant determinant of RER dynamics.

It is important to stress at the outset that since we employ a standard Heckscher-Ohlin production structure, the long-run real exchange rate – when all the sluggishness has been worked through – is determined by production conditions alone. Consequently, the elasticity of labor supply

\(^8\) Measured in widely used metric half-life, meaning in response to a shock, how long does it take for the RER to come back to half of the initial change.
has no impact on the long-run equilibrium level of the real exchange rate. But it does have profound implications for the short-run dynamics, such as the short-run volatility and the speed of convergence. Accordingly, these transitional aspects are the elements upon which we focus.

Previous work, based on an inelastic labor supply, has shown that the direction of the short-run adjustment of the real exchange rate depends upon the source of the structural change, while its magnitude is much larger for shocks occurring in the nontraded sector, where the scope for quantity adjustment is more limited. While the same remains true here, the relative contribution of the labor supply adjustment to the total adjustment in the real exchange rate is larger when the structural change occurs in the traded sector. This is because with the price of the traded good determined abroad, more of the adjustment is pushed onto quantities, including the labor supply.

Endogenizing the labor supply changes the short-run dynamics of the RER substantially in several crucial, and quantitatively significant, ways. We compare the RER responses to increases in government expenditure (demand shocks) and productivity (supply shocks), paying particular attention to the role of key taste and production parameters. The main conclusions are the following, where the comparison is with a model involving sectoral adjustment costs and inelastic labor supply.

First, the elastic labor substantially reduces the short-run decline of the RER, in response to an increase in government expenditure on the traded good. In contrast, the elastic labor supply exacerbates the short-run increase of the RER, following an increase in government expenditure on the nontraded good. This difference reflects the fact that in both cases the wealth effect operates in the same direction, offsetting the decline in the RER in the former case, and reinforcing the increase in the latter. More specifically, the additional taxes necessary to finance the additional government expenditures reduce private wealth, thus increasing its marginal utility and inducing agents to increase their labor supply. Being forward-looking, agents know that the increased labor supply will increase future output, against which they will borrow in order to smooth consumption. Thus the total demand for both the traded and nontraded good will be greater with elastic labor supply compared to that for inelastic labor supply. These patterns are uniform across variations in the elasticities of substitution between consumption and leisure in utility, and capital and labor in production, although the relative importance of the elastic labor supply effect does vary. To the
extent that in more open economies governments spend relatively more on traded goods, this implies less short-run volatility in the RER, consistent with the empirical evidence obtained by Hau (2002).

Second, depending upon the elasticity of substitution between consumption and leisure in utility, the elastic labor may either reduce or increase the short-run increase of the RER, following an increase in the productivity of the traded sector. Basically it depends upon the size of the wealth effect stemming from the productivity increase, which has a dampening effect on labor supply, relative to the real wage effect, which has the opposite effect. On the other hand, the short-run decline in the RER resulting from an increase in the productivity of the nontraded sector is increased with an elastic labor supply.

Third, endogenizing the labor supply significantly affects the time path of the rate of convergence of the RER. Elastic labor supply always reduces the long-run speed of convergence. The fact that some of the long-run adjustment to a structural change is borne by labor supply allows the exchange rate to adjust more slowly. But the effect of elastic labor supply on the short-run convergence speed depends upon the source of the shocks generating the dynamics. For demand shocks originating in the traded sector the labor supply effect tends to reduce the rate of convergence at all stages during the transition; for demand shocks in the nontraded sector it increases the short-run rate of convergence. For productivity increases in the traded sector the short-run labor supply effect is highly sensitive to the elasticity of substitution between consumption and leisure in utility; in contrast, for productivity increases shocks in the nontraded sector, it reduces the short-run rate of convergence, albeit it slightly.

The rest of the paper is structured as follows. Section 2 sets out the analytical framework, which we analyze numerical simulations, the parameterization of which is discussed in Section 3. Section 4 discusses the dynamics of the real exchange rate, emphasizing how it is influenced by the endogeneity of labor supply, while Section 5 concludes.

2. The Analytical Framework

The model we employ is an extension of Morshed and Turnovsky (2004) to include elastically supplied labor, and accordingly our description can be brief. We consider a small open
economy, inhabited by a single representative agent, who is endowed with a fixed unit of time that can be allocated to leisure, \( l \), or supplied as labor, which he sells at the competitive wage. The agent produces a traded good, \( Y_T \), (taken to be the numeraire) and a nontraded good, \( Y_N \), using a quantity of capital, \( K \), and labor, \( L \), by means of conventional neoclassical production functions. The agent allocates his labor between these two production processes and consumes both the traded and nontraded good. While the former is used only for consumption (either private or public), the latter may be either consumed or accumulated as a capital good, to which it may be converted without incurring any adjustment costs. This latter assumption is made because, in order to focus on intersectoral adjustment costs, which are essential to generating gradual RER dynamics, we wish to keep other adjustment processes as simple as possible.

The agent also accumulates net foreign bonds, \( B \), that pay a given world interest rate \( r \), in accordance with the agent’s instantaneous budget constraint:

\[
\dot{B} = F(K_T, L_T) - C_T + \sigma[H(K_N, L_N) - C_N - I] - T_L + rB \tag{1a}
\]

where \( C_T \) and \( C_N \) are the agent’s consumption of traded goods and nontraded goods, respectively; \( F(K_T, L_T) \) and \( H(K_N, L_N) \) are the sectoral production functions and factor allocations; \( \sigma \) is the relative price of nontraded goods to traded goods; \( I \) denotes new investment, and \( T_L \) denotes lump-sum taxes.\(^9\)

We further assume that the capital stock does not depreciate and that moving it across sectors involves adjustment costs. Only nontraded new output can be converted into capital, and once it is set aside as capital in the nontraded sector, it requires extra resources to transform it into a form that is suitable for use in the traded sector. Accordingly, capital accumulation is described by:

\[
\dot{K}_T = X \tag{1b}
\]

\[
\dot{K}_N = I - X \left( 1 + \frac{h}{2} \frac{X}{K_N} \right) \tag{1c}
\]

where \( X \) is the amount of capital transferred from the nontraded to the traded sector, and

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\(^9\) Assuming that purchasing power parity holds for traded goods, \( \sigma \) also measures the real exchange rate.
identifies the amount of nontraded output available for investment as being the amount of nontraded output remaining after both private consumption, \( C_N \), and government purchases, \( G_N \), have been met. In order to provide \( X \) units of capital to the traded sector, the amount of capital in the nontraded sector must be reduced by more than \( X \). \(^{10}\) This excess amount, \( hX^2/2K_N > 0 \), represents the sectoral adjustment (retrofit) costs. \(^{11}\)

Summing (1b) and (1c), the total rate of capital accumulation in the economy, \( \dot{K} \), is

\[
\dot{K} = \dot{K}_T + \dot{K}_N = I - \frac{hX^2}{2K_N} \quad (1e)
\]

where the last term in (1e) denotes the loss in capital due to sectoral movements. In the absence of sectoral adjustment costs, (1e) reduces to the standard aggregate capital accumulation relationship, \( \dot{K} = I \). Finally, labor is perfectly mobile across sectors and the labor market always clears, so that the following equation holds at all times. \(^{12}\)

\[
L_T + L_N + l = 1 \quad (1f)
\]

The agent’s decisions are to choose his consumption levels \( C_T, C_N \), labor allocation and leisure, \( L_T, L_N, l \), the capital allocation decisions \( K_T \) and \( K_N \), the rate of investment \( I \), and his rate of accumulation of traded bonds to maximize the following intertemporal utility function

\[
\int_0^\infty U(C_T, C_N, I)e^{-\beta t} dt \quad (2)
\]

subject to the constraints (1a) – (1d), (1f), and given initial stocks \( K_T(0) = K_{T,0}, K_N(0) = K_{N,0} \), and \( B(0) = B_0 \). The instantaneous utility function is assumed to be concave and all goods are assumed to

\(^{10}\) As usual, the formulation permits negative aggregate investment. The usual interpretation of this is that the agent is permitted to consume his capital stock or sell it in the market for new output.

\(^{11}\) Morshed and Turnovsky (2004) show how varying \( h \) from 0 through \( \infty \) enables them to encompass the standard Heckscher-Ohlin technology at one extreme and the sector-specific capital model at the other. Grossman (1983) has a similar index of capital mobility measured by the percentage loss in efficiency that is incurred in transforming the marginal unit of capital.

\(^{12}\) The assumption that labor can move costlessly between sectors, while less objectionable than perfect sectoral capital mobility, is also restrictive, since in reality this will involve labor retraining costs; see Dixit and Rob (1994). The presence of sunk costs in their model generates hysteresis in the movement of labor across sectors.
be normal. The agent’s rate of time preference, $\beta$, is taken to be constant. The optimality conditions are:

$$U_T(C_T, C_N, l) = \lambda$$  \(3a)$$

$$U_N(C_T, C_N, l) = \lambda \sigma$$  \(3b)$$

$$U_i(C_T, C_N, l) = \lambda F_L(K_T, L_T)$$  \(3c)$$

$$F_L(K_T, L_T) = \sigma H_L(K_N, L_N)$$  \(3d)$$

$$\frac{X}{K_N} = \frac{(q_1 - q_2)}{q_2 h}$$  \(3e)$$

$$\sigma = q_2$$  \(3f)$$

$$\beta - \frac{\lambda}{\beta} = r$$  \(3g)$$

$$\frac{F_K + \dot{q}_1}{q_1} = r$$  \(3h)$$

$$H_K + \frac{hX^2}{2K_N^2} + \frac{\dot{q}_2}{q_2} = r$$  \(3i)$$

where $\lambda, \lambda_1', \lambda_2'$ denote the shadow values of wealth in the form of internationally traded bonds, capital in the traded sector, and capital in the nontraded sector, respectively so that $q_1 \equiv \lambda_1'/\lambda$, $q_2 \equiv \lambda_2'/\lambda$ may be interpreted as the market prices (values) of the traded and nontraded capital respectively. 13 In addition, the following transversality conditions apply:

$$\lim_{t \to \infty} \lambda Be^{-\beta t} = \lim_{t \to \infty} q_1 \lambda K_T e^{-\beta t} = \lim_{t \to \infty} q_2 \lambda K_N e^{-\beta t} = 0$$  \(3j)$$

Equations (3a)-(3d) are standard static efficiency conditions and require no further comment. Equation (3e) determines the rate at which capital is moving between the sectors. Capital flows from the sector where it is less valued to the sector where it is more valued, at a rate that is inversely

\[13 \text{Thus, } q_1, q_2 \text{ are unit-free (like the Tobin } q)\]
related to the size of the adjustment cost parameter, \( h \). Since nontraded output can be either converted into capital or consumed, in equilibrium the agent is indifferent between these two uses of new output. This yields the equality of the marginal utility of consumption of nontraded goods, \( \lambda \sigma \), and the shadow value of capital, \( q_z \lambda \), in the nontraded sector, and reduces to equation (3f).

The remaining three equations are intertemporal efficiency conditions. Equation (3g) equates the rate of return on consumption to the rate of return on traded bonds. To obtain a well-defined interior steady-state equilibrium, we require \( \beta = r \) which implies that \( \lambda = 0 \) for all \( t \), so that the marginal utility, \( \lambda \), remains constant at all times, i.e., \( \lambda = \overline{\lambda} \). Equations (3h) and (3i) equate the rates of return on traded and nontraded capital to the rate of return on traded bonds. Both include the “payout rate” (the appropriately valued marginal physical product) plus the rate of capital gain. In addition, since increasing the stock of nontraded capital reduces the adjustment costs, this comprises a third component of the rate of return to nontraded capital.

The government in this economy is passive. It simply raises lump-sum taxes to finance its expenditures on the traded and nontraded good, \( G_T \) and \( G_N \), respectively, in accordance with its budget constraint \( T_L = G_T + \sigma G_N \). For simplicity, we assume that the utility government spending provides is additively separable from that yielded by private consumption, so that without any loss of generality it can be ignored.

### 2.1 Macroeconomic Equilibrium

The macroeconomic equilibrium is obtained as follows. First, we solve equations (3a)-(3d) and (1f) for traded and nontraded consumption, \( C_T \) and \( C_N \), sectoral labor allocations \( L_T, L_N \), and leisure, \( l \), \( C_T = C_T(\overline{\lambda}, \sigma, K_T, K_N) \), \( C_N = C_N(\overline{\lambda}, \sigma, K_T, K_N) \), \( L_T = L_T(\overline{\lambda}, \sigma, K_T, K_N) \), \( L_N = L_N(\overline{\lambda}, \sigma, K_T, K_N) \), \( l = l(\overline{\lambda}, \sigma, K_T, K_N) \). The macroeconomic equilibrium can thus be summarized by the following autonomous system in the four variables, \( K_T, K_N, \sigma, X \)

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14 This assumption is standard in intertemporal models of small open economies, in which we assume agents have a constant rate of time preference and face a fixed world interest rate. Although it is not particularly appealing, it is not a serious limitation for the issue we are addressing. Alternative ways of breaking this “knife-edge condition” are discussed by Turnovsky (2002) and Schmitt-Grohé and Uribe (2003).

15 In the absence of sectoral adjustment costs, \( h = 0 \), implying \( q_1 = q_2 = \sigma \). Substituting these conditions into (3g) and (3h), the latter reduce to the standard static efficiency condition for capital allocation, \( F_k = \sigma H_k \).
\[ \dot{K}_r = X \]  
\[ \dot{K}_N = H \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) - C_N(\bar{\lambda}, \sigma, K_T, K_N) - X \left( 1 + \frac{hX}{2K_N} \right) - G_N \]  
\[ \dot{\sigma} = \sigma \left( r - H_K \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) - \frac{hX^2}{2K_N^2} \right) \]  
\[ \dot{X} = \left( \frac{H \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) - C_N(\bar{\lambda}, \sigma, K_T, K_N) - G_N}{K_N} + H_K \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) \right) X \]  
\[- \frac{X^2}{2K_N} - \frac{K_N}{h\sigma} \left( F_K \left( K_T, L_T(K_T, K_N, \sigma, \bar{\lambda}) \right) - \sigma H_K \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) \right) \]  

\[ \tilde{B} = F \left( K_T, L_T(K_T, K_N, \sigma, \bar{\lambda}) \right) - C_T \left( K_T, K_N, \sigma, \bar{\lambda} \right) + rB - G_T \]  

### 2.2 Steady State and Equilibrium Dynamics

The economy reaches steady state when \( \dot{K}_r = \dot{K}_N = \dot{\sigma} = \dot{X} = \dot{B} = 0 \), implying further that in steady state, \( X = 0 \). Imposing these conditions and yields the steady-state relationships

\[ H_K \left( \tilde{K}_N, L_N(\tilde{K}_N, \tilde{\sigma}, \bar{\lambda}) \right) = r \]  
\[ \frac{1}{\tilde{\sigma}} F_K \left( \tilde{K}_T, L_T(\tilde{K}_T, \tilde{\sigma}, \bar{\lambda}) \right) = r \]  
\[ F \left( K_T, L_T(K_T, K_N, \sigma, \bar{\lambda}) \right) = C_T \left( K_T, K_N, \sigma, \bar{\lambda} \right) + G_T - r\tilde{B} \]  
\[ H \left( K_N, L_N(K_T, K_N, \sigma, \bar{\lambda}) \right) = C_N \left( K_T, K_N, \sigma, \bar{\lambda} \right) + G_N \]

Using the homogeneity of the production functions, equations (3d), (4c’) and (4d’) can be expressed in the standard intensity form

\[ f_\ell \left( \tilde{K}_T/\tilde{L}_T \right) = \tilde{\sigma} h_\ell \left( \tilde{K}_N/\tilde{L}_N \right) \]  
\[ h_K \left( \tilde{K}_N/\tilde{L}_N \right) = r \]
These three equations illustrate the well known result that for the Heckscher-Ohlin technology, the long-run sectoral capital-labor ratios, $\bar{K}_T/\bar{L}_T$, $\bar{K}_N/\bar{L}_N$, and the real exchange rate, $\bar{\sigma}$, are jointly determined by the production sector alone. In particular, the long-run real exchange rate is independent of demand conditions, including the labor supply conditions, as determined by the agent’s labor-leisure choice.\textsuperscript{16}

Linearizing (4a) – (4d) around the steady state (denoted by tildes), the dynamics of $K_T$, $K_N$, $\sigma$, and $X$ can be approximated by

$$
\begin{pmatrix}
\dot{K}_T \\
\dot{K}_N \\
\dot{\sigma} \\
\dot{X}
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 & 1 \\
a_{21} & a_{22} & a_{23} & -1 \\
a_{31} & a_{32} & a_{33} & 0 \\
a_{41} & a_{42} & a_{43} & H_K
\end{pmatrix}
\begin{pmatrix}
K_T - \bar{K}_T \\
K_N - \bar{K}_N \\
\sigma - \bar{\sigma} \\
X - \bar{X}
\end{pmatrix}
$$

where

$$
a_{21} = H_L \frac{\partial L_N}{\partial K_T} - \frac{\partial C_N}{\partial K_T}; a_{22} = H_L \frac{\partial L_N}{\partial K_N} - \frac{\partial C_N}{\partial K_N}; a_{23} = H_L \frac{\partial L_N}{\partial \sigma} - \frac{\partial C_N}{\partial \sigma};
$$

$$
a_{31} = -\sigma H_{KL} \frac{\partial L_N}{\partial K_T}, a_{32} = -\sigma \left( H_{KK} - H_{KL} \frac{\partial L_N}{\partial K_N} \right),
$$

$$
a_{33} = -\sigma H_{KL} \frac{\partial L_N}{\partial \sigma};
$$

$$
a_{41} = -\frac{K_N}{h\sigma} \left[ F_{kk} + F_{KL} \frac{\partial L_T}{\partial K_T} - \sigma H_{KL} \frac{\partial L_N}{\partial K_T} \right]; a_{42} = -\frac{K_N}{h\sigma} \left[ F_{KL} \frac{\partial L_T}{\partial K_N} - \sigma \left( H_{KK} + H_{KL} \frac{\partial L_N}{\partial K_N} \right) \right];
$$

$$
a_{43} = -\frac{K_N}{h} \left[ -\frac{F_{KL}}{\sigma^2} \frac{\partial L_T}{\partial \sigma} - H_{KL} \frac{\partial L_N}{\partial \sigma} \right].
$$

Equation (5) describes a fourth-order linear dynamic system, and by examining its characteristic equation we can establish that there are two eigenvalues having positive real parts and two with negative real parts, implying that the equilibrium is a saddlepoint. We assume that the two capital stocks, $K_T$ and $K_N$, are constrained to move sluggishly, while the relative price, $\sigma$, and the

\textsuperscript{16} There is an extensive empirical literature examining whether the long-run real exchange rate is determined primarily by demand shocks or supply shocks. As is often the case, the evidence is mixed. Interestingly, Alexius (2001) find that, consistent with the dependent economy model, productivity changes are the key determinants of long-run exchange rate movements in the four Nordic countries he studies.
rate of intersectoral capital transfer, \( X \), are free to jump instantaneously, so that the equilibrium yields a unique stable saddlepath.

To solve this model, we follow the procedure proposed by Turnovsky (1997) where we linearize the current account equation around the steady-state and then substitute the linear solutions obtained from equations (4a-4d). Since the intertemporal solvency condition derived from the current account dynamics, the steady-state equilibrium, and the eigenvalues determining the transitional dynamics about the equilibrium are simultaneously determined, we have solved for the complete equilibrium dynamics recursively.

2.3 Rate of Convergence

The key issue we wish to discuss concerns the rate of convergence of the real exchange rate, \( \sigma(t) \), following some structural or policy shock. Denoting the stable adjustment path of \( \sigma(t) \) by

\[
\sigma(t) - \bar{\sigma} = D_1 v_{31} \mu_1^t + D_2 v_{32} \mu_2^t
\]

where \( \mu_2 < \mu_1 < 0 \) are the stable eigenvalues, \( (1, v_{2i}, v_{3i}, v_{4i}) \) is the normalized eigenvector associated with the eigenvalue \( \mu_i \), and \( D_1, D_2 \) are constants determined by the structural change, we can define the rate of convergence at time \( t \), \( \kappa(t) \), by

\[
\kappa(t) \equiv \frac{\dot{\sigma}}{\sigma - \sigma(t)} = \frac{D_1 v_{31} \mu_1^t}{D_1 v_{31} \mu_1^t + D_2 v_{32} \mu_2^t} (-\mu_1) + \frac{D_2 v_{32} \mu_2^t}{D_1 v_{31} \mu_1^t + D_2 v_{32} \mu_2^t} (-\mu_2)
\]

This is a time-varying weighted average of the magnitudes of the two stable eigenvalues. Initially,

\[
\kappa(0) = \left( \frac{D_1 v_{31}}{D_1 v_{31} + D_2 v_{32}} \right) (-\mu_1) + \left( \frac{D_2 v_{32}}{D_1 v_{31} + D_2 v_{32}} \right) (-\mu_2)
\]

and asymptotically, \( \kappa(t) \rightarrow \tilde{\kappa} \equiv -\mu_1 > 0 \), the larger of the two stable eigenvalues.

3. Parameterization

17 In writing the solution for \( \sigma(t) \) in this way, we are assuming that the four dynamic variables are ordered as in (5).

18 This definition is chosen so as to ensure that as long as \( \sigma(t) \) is approaching \( \bar{\sigma} \), the rate of convergence \( \kappa(t) > 0 \). Values of \( \kappa(t) < 0 \) correspond to movements away from equilibrium, i.e. divergence. For further discussion of this measure see Eicher and Turnovsky (1999).
The linearized equilibrium system, (5), is too complex for formal analysis. It is necessary to resort to numerical simulations, for which we adopt the following utility and production functions:

**Utility Function:**

\[ U = \frac{1}{\gamma} \left[ aC^{-\rho} + (1 - a)L^{-\rho} \right]^{\gamma / \rho} \quad 0 < a < 1, \quad -\infty < \gamma < 1 \quad (7a) \]

\[ C = C_T^\theta C_N^{1-\theta} \quad 0 < \theta < 1 \quad (7b) \]

**Production Functions:**

\[ F(K_T, L_T) = \varphi [mK_T^{-\alpha} + (1 - m)L_T^{-\alpha}]^{-1/\alpha} \quad \varphi > 0, 0 < m < 1 \quad (8a) \]

\[ H(K_N, L_N) = \psi [nK_N^{-\delta} + (1 - n)L_N^{-\delta}]^{-1/\delta} \quad \psi > 0, 0 < n < 1 \quad (8b) \]

Utility is of a nested form. First, the two consumption goods interact to yield the composite consumption good in (7b), which then interacts with leisure to provide overall utility. Preferences are summarized by four parameters: (i) \( \theta \), which measures the relative importance in utility of traded versus nontraded goods; (ii) \( a \), which parameterizes the relative importance of overall consumption versus leisure; (iii) \( \eta \equiv 1/(1 + \rho) \), the *intratemporal* elasticity of substitution between consumption and leisure; (iv) \( \equiv 1/(1 - \gamma) \), the *intertemporal* elasticity of substitution.

Production in both sectors is represented by a Constant Elasticity of Substitution (CES) production function, where \( s_T \equiv 1/(1 + \alpha) \) and \( s_N \equiv 1/(1 + \delta) \) define the (constant) elasticities of substitution for production in the two sectors, respectively. The Cobb-Douglas production functions are obtained by letting \( \alpha \to 0, \delta \to 0 \). The coefficients \( \varphi \) and \( \psi \) parameterize the productivity in the traded and nontraded goods sector respectively, while \( m \) and \( n \) parameterize the respective capital intensities in the two sectors. Since the behavior of the economy is sensitive to the relative sectoral capital intensities, we will identify two benchmark equilibria, depending upon whether the traded sector is more capital intensive than the nontraded sector \( (m > n) \) or vice versa \( (n > m) \). Taken together, the flexibility of these chosen functional forms enables us to address various aspects of the role of flexibility of labor supply.

Table 2.A reports base parameter values, while Table 2.B summarizes the corresponding key steady-state equilibrium ratios. These benchmark cases are based on Cobb-Douglas sectoral production functions and utility function. They are therefore identical to those chosen by Morshed.
and Turnovsky (2004) where they are explained and discussed in detail. Most of the chosen parameter values are standard and non-controversial. As we also discuss in the earlier paper, the resulting steady-state equilibrium values are all plausible. The quantities pertaining to the breakdown between the traded and non-traded sector are generally less well documented, but these have been derived in detail as averages for some 30 trading economies; see Morshed and Turnovsky (2004). Since the evidence on the size of sectoral adjustment costs is so sparse and indirect, Morshed and Turnovsky (2004) argue, following Caballero and Engel (1999) and Ramey and Shapiro (2001), that the size of the sectoral adjustment costs, involving retrofitting, should be larger than those for new aggregate investment, where \( h \) is typically within the range 10-15. On that basis they set \( h = 30 \) as a benchmark, an assumption we retain.

The new parameter introduced by the elastic labor supply is the relative weight \( a = 0.30 \) on consumption in utility. For the Cobb-Douglas utility function this implies an overall allocation of time to labor of around 0.299 when the traded good sector is more capital intensive, while it is 0.281 when nontraded good in more capital intensive, both values being generally consistent with empirical evidence; see Cooley (1995).

The two sets of equilibria reported in Table 1.B correspond to the two sets of productivity elasticities, \( m = 0.35, n = 0.25 \), and \( m = 0.25, n = 0.35 \), respectively. The reason for keeping these elasticities within this narrow range is that they reflect the share of capital in the respective output of that sector. Since both the traded and nontraded sectors themselves represent substantial aggregates, we would not expect their production functions to differ too dramatically from the overall aggregate, for which the elasticity of capital typically is in the above ranges.

The aggregate capital output ratio is 3.74 when traded sector is more capital intensive while it is slightly higher (4.83) when nontraded sector is more capital intensive. About 41% (44%) of total output is produced in the traded sector when traded (nontraded) sector was more capital intensive. Government expenditures are chosen to be in the range of 11-13% of traded output and 31-40% of nontraded output, which are plausible in light of average ratios calculated using data from 30

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19 For example, \( \gamma = -1.5 \) implies an intertemporal elasticity of substitution of 0.4 and is well in the range of empirical estimates as summarized by Guvenen (2006), while the exogenous world interest rate of 6% is plausible as well.

20 We did experiment with other values of \( h \) but our results remain largely unchanged.
countries by Morshed and Turnovsky (2004).\textsuperscript{21}

Subsequent simulations we undertake involve extensive sensitivity analysis. These are presented in the form of a grid where we allow the sectoral elasticities of substitution, $s_r, s_N$ to range between 0.8 (low) and 1.1 (high) and the elasticity of substitution in utility function, $\eta$, to vary between 0.5 and 2.\textsuperscript{22,23}

Table 3 reports the steady-state supply of labor. From this table we observe that, irrespective of the sectoral capital intensity, the steady-state labor supply is decreasing in both the elasticity of substitution between capital and labor in production, and in the elasticity of substitution between leisure and consumption in utility. This is because labor yields disutility and the more flexibility agents have to substitute away from it, the more they will do so. Since we assume perfect sectoral mobility of labor, and labor allocation will be one of the equilibrating forces, the pattern of labor supply will turn out to be crucial to the RER dynamics.

4. Dynamics of the Real Exchange Rate

In this section we discuss the dynamics of the real exchange rate in response to four structural changes: (i) An increase in $G_r$ from 0.045 to 0.06; (ii) an increase in $G_N$ from 0.12 to 0.15; (iii) an increase in productivity of the traded sector, $\varphi$, from 1.5 to 1.65; (iv) an increase in the productivity of the nontraded sector, $\psi$, from 1 to 1.1. In all cases the changes are relatively small, thus ensuring that linearizing the system to analyze the transitional dynamics does not lead to unacceptable approximation errors; see Atolia, Chatterjee, and Turnovsky (2010).

4.1 Short-run Volatility

Table 4 reports the short-run and long-run elasticities of the real exchange rate with respect to increases in government expenditure on traded and nontraded output (demand shocks) and

\textsuperscript{21} The implied overall share of government expenditure in GDP is 0.24 if $m = 0.35, n = 0.25$ and 0.18, if these sectoral intensities are reversed. Both ratios are well within the observed range computed by Morshed and Turnovsky (2004).

\textsuperscript{22} There is much less information on the elasticity of substitution between leisure and consumption in utility. While the Cobb-Douglas form of the utility function, with the implied value $\eta = 1$ is standard, estimates of $\eta = 0.4$ have also been obtained; see e.g. Stern (1976) for an early well known example.

\textsuperscript{23} In the paper we restrict our consideration for variations in $a$ to 0.30. Results for other values of $a$ are available from the authors.
increases in aggregate productivity in the traded and nontraded sectors (supply shocks) for both cases of relative sectoral capital intensities. Our objective is to identify the role of the elastic labor supply. Since the equilibrium labor supply varies for each combination of the parameters \((s_T, s_N, \eta)\) [see Table 3], in each case we first compute the corresponding endogenous supply of labor, and then normalize the labor supply at that same level in the case that it is taken to be in fixed supply.

All the long-run responses of the RER confirm their known analytical properties, which can also be directly verified from (3d’), (4c”), (4d”). These include: (i) the elasticity with respect to demand shocks is zero; (ii) the elasticity with respect to \(\phi\) is one, (iii) the elasticity with respect to \(\psi\) is independent of the labor supply conditions.

In the short run, labor supply conditions are seen to play an important role in determining the response of the real exchange rate, though this will vary with the shock. In each case, we begin with the benchmark parameterization \((s_T = s_N = \eta = 1)\) which are indicated in bold type.

*Increase in \(G_T\)*: From Table 4.A we see that the decline in the RER is dramatically reduced with elastic labor supply. In the case where the traded sector is more capital intensive, the decline is reduced by over 50%; from -0.030 with inelastic labor supply to -0.014. When factor intensities are reversed it is moderated even more, from -0.041 to -0.013. This pattern is consistent across the table, although there are relative differences. Specifically, the reductions in the real exchange rate due to the elastic labor supply become relatively smaller as the elasticities of substitution in production and utility increase.

Overall, the elastic labor supply will tend to reduce the short-run volatility in the real exchange rate which originates with increases in government spending on the traded good, doing so by significant amounts. With inelastic labor supply the increase in government demand for the traded output will generate an increase in the relative price of traded output, i.e. a decline in \(\sigma(0)\), causing a decline in private consumption of that good. But with elastically supplied labor a second effect is operative. In addition, the increase in lump-sum taxes necessary to finance the additional government expenditure will raise the marginal utility of wealth inducing agents to increase their supply of labor, thereby increasing their future income, and the forward looking agents smooth their consumption by borrowing from the future higher income and so their demand for both traded and
nontraded goods increases. This will reduce the decline in private demand for the traded good, thereby mitigating the decline in the relative price of the nontraded good.

*Increase in $G_N$: In contrast, we see that the increase in the RER in response to an increase in the government expenditure on nontraded goods, is substantially increased with elastic labor supply. In the benchmark case the short run elasticity increases from 0.151 to 0.192 when the traded sector is more capital intensive, with the increase being even larger (0.125 to 0.180) when the sectoral factor intensities are reversed. In contrast to $G_T$, the relative importance of the labor supply component increases with the elasticity of substitution in production, but declines with the elasticity of substitution between consumption and leisure in utility.

In this case the elastic labor supply will tend to increase the short-run volatility in the real exchange rate due to increases in government spending on the nontraded good. The reason is that with inelastic labor supply the increase in overall demand for the nontraded good generated by the government expenditure will immediately increase its relative price $\sigma(0)$, causing a decline in the private consumption of that good. But with elastically supplied labor there is the additional wealth effect. The increase in lump-sum taxes necessary to finance the additional government expenditure will raise the marginal utility of wealth inducing agents to increase their supply of labor, thereby increasing their future income, and the consumption smoothing of the forward looking agents increases their demand for both traded and nontraded goods. This will increase the private demand for the nontraded good, thereby increasing its relative price further.

These results are consistent with the empirical findings of Hau (2002), who found that the open economy has a less volatile real exchange rate. In this case, the more open economy the relatively more government spending is on the traded good and thus the less volatile is its real exchange rate for the reasons just discussed.

*Increase in $\phi$: Focusing on the benchmark technology in Table 4.C we see that the short-run increase in the RER resulting from an increase in the productivity of the traded good is reduced somewhat with elastic labor supply. In the case where the traded sector is more capital intensive, the increase is reduced slightly from 1.029 when labor is supplied inelastically to 1.014. When factor intensities are reversed it is reduced even more, from 1.039 to 1.013. However, this pattern is
sensitive to the elasticity of substitution between leisure and consumption in utility. For low values of $\eta$, the increases in the real exchange rate due to the elastic labor supply become relatively smaller as the elasticity of substitution in production increases. For high values of $\eta$ the opposite is true. Moreover, while for low values of $\eta$ the elastic labor supply tends to reduce the short-run increase in the RER, for high values of $\eta$ a flexible labor supply will tend to exacerbate the short-run increase in the RER.

Thus the impact of the elastic labor supply on the short-run volatility of RER stemming from an increase in the aggregate productivity of the traded sector is ambiguous. With inelastic labor supply a given increase in the productivity of the traded sector will lead to a more than proportionate increase in the RER. While this causes a decline in private consumption of the nontraded good, this is offset by the wealth effect resulting from the increase in productivity. With elastically supplied labor the productivity increase generates two effects. First, it increases the productivity of labor, increasing the real wage, and stimulating labor supply. At the same time, the increase in wealth due to the increase in productivity and enhanced future output will reduce labor supply. These responses are offsetting and the dominant effect depends upon the elasticity of substitution between consumption and leisure, as the numerical simulations suggest.

Increase in $\psi$: From Table 4.D we see that the short-run decline in the RER resulting from an increase in the productivity of the nontraded good is increased with elastic labor supply. Focusing on the benchmark technology, in the case where the traded sector is more capital intensive, the decline is increased from -0.401 with inelastic labor supply to -0.476. When factor intensities are reversed the reduction is even bigger, from -0.371 to -0.499. This pattern is consistent across the table, although there are relative differences. As with $G_N$, the relative importance of the labor supply component increases with the elasticity of substitution in production, but declines with the elasticity of substitution between consumption and leisure in utility. In this case the elastic labor supply will tend to increase the short-run volatility in the real exchange rate due to productivity increases in the nontraded sector. The reasoning parallels that of other cases, with the impact of the additional wealth on labor supply being the key element.
4.2 Speeds of Convergence

Table 5 reports both short-run and long-run (asymptotic) rates of convergence of the real exchange rate for both fixed and elastic labor supply. The difference between the short-run and long-run rates highlights the fact that the speeds of convergence vary enormously during the transition. The long-run rates of convergence are slower with elastic labor supply than they are for fixed labor supply, irrespective of the sectoral capital intensities and elasticity of substitution between capital and labor. Moreover, for a larger elasticity of substitution between consumption and leisure, the long-run rate of convergence increases under fixed labor supply while it decreases under variable labor supply. Thus the difference between the asymptotic rates of convergence with fixed labor supply versus elastic labor supply increases with the elasticity of substitution between consumption and leisure. This is true for both demand and supply shocks.

In contrast, the comparative short-run rates of convergence depend on the source of the shocks. When the shock is due to an increase in government expenditure on the traded good, the short-run rates of convergence obtained for elastic labor supply are slower than those under fixed labor supply. Indeed, for larger values of the elasticity of substitution between consumption and leisure (\(\eta = 1, 2\)), the RER initially diverges before it starts converging. On the other hand, the short-run rates of convergence in response to an increase in government expenditure on the nontraded good are always faster with elastic labor supply than under fixed labor supply.

The same is true for an increase in the aggregate productivity of the traded sector except in the case of unitary elasticity of substitution between consumption and leisure, the short-run rates of convergence is smaller (yields negative rate of convergence under variable labor supply). The aggregate productivity increase in nontraded goods, however, yields slower short-run rates of convergence. In this case, the gap between the short run rates of convergence under fixed vs variable labor supply becomes more pronounced with a larger elasticity of substitution between consumption and leisure.24

24 We have also computed the half-lives of the adjustments in response to the various structural changes in order to correlate them with the corresponding short-run changes in labor supply. But since each change has a different impact, both quantitative and qualitative, on short-run employment it is difficult to relate these correlations to the aggregate correlation of 0.40, calculated from the empirical data, which reflects a combination of these conflicting effects.
4.3 Transitional Dynamics

Figure 1 compares the dynamics of the RER under fixed labor versus elastic labor supply in response to the two types of government expenditure (demand shocks) and the two forms of productivity increase (supply shocks). These figures are drawn for the benchmark specification of the production functions and utility function \((s_r = s_s = \eta = 1)\) and traded sector is more capital intensive \((m = 0.35, n = 0.25)\). To preserve comparability we first calculated the steady-state labor supply when labor is supplied elastically and then set the labor supply at that same level in the alternative case where it is supplied inelastically. This yields the same initial level of the RER for both the fixed labor and endogenous labor models. In addition, we also assume the same sectoral adjustment costs \((h = 30)\) for the inter-sectoral movement of capital\(^{25}\).

The figure illustrates how the introduction of elastic labor supply can change the entire dynamic adjustment significantly, depending upon the shock. Fig. 1.A illustrates how, in response to an increase in government expenditure on traded good, the presence of elastic labor supply moderates the initial decline in the real exchange rate and slows down the rate of convergence over the entire path. Fig. 1.B shows the contrasting effect of the elastic labor supply when the government increases its expenditure on the nontraded good. The short-run appreciation of the real exchange rate is exacerbated, the rate of convergence initially increases, but slows down over time. Fig. 1.C illustrates how the elastic labor supply moderates the initial appreciation of the real exchange rate following an increase in the productivity of the traded sector. In the short run it actually diverges, before converging at a slower rate. Finally Fig. 1.D illustrates how the elastic labor supply increases the initial decline in the RER. Thereafter, it adjusts at a slower rate than under a fixed labor supply, in both cases overshooting the new long-run steady state value of 1.712. The initial more rapid adjustment under fixed labor supply causes the exchange rate to overshoot more in that case, after which it continues to converge at a faster rate, as it corrects this mis-

\(^{25}\) Morshed and Turnovsky (2004) showed that the presence of sectoral adjustment costs generates transitional dynamics of the RER even in response to demand shocks, while the role of elasticity of substitution in production is also discussed by Morshed and Turnovsky (2006). However, in both these papers labor supply was assumed to be fixed.
5. Conclusions

The real exchange rate is a key economic factor for any international trading economy. Empirical evidence has suggested that the flexibility of labor supply is related to the dynamic adjustment of the real exchange rate. In this paper we have investigated this relationship in a two-sector dependent economy model. While, it is known that the long-run equilibrium real exchange rate is independent of the elasticity of labor supply in this type of framework, our analysis confirms that the nature of the labor supply can be a crucially important determinant of its short-run dynamics. The extent to which this is so depends to some degree on the source of the underlying structural change that is driving the dynamics of the real exchange rate.

The mechanism we have analyzed, the flexibility of labor supply, may provide some support for some of the empirical evidence on real exchange rate dynamics. Consider, for example, Cheung and Lai’s (2000) study suggesting that the rate of convergence of the real exchange rate is slower for developed countries than for developing countries. While one explanation may be the larger elasticity of substitution between labor and capital in developed countries, as argued by Morshed and Turnovsky (2006), the present paper suggests an alternative reason. Being wealthier, developed economies enjoy a greater elasticity of labor supply, which as we have seen is generally associated with a slower convergence of the real exchange rate.

Our results may also offer some insight into the observation that developing countries experience greater volatility of the real exchange rate; see Hausmann, Panizza, and Rigobon (2006). Given that developing countries have less flexible labor markets, and are likely more trade dependent, the larger short-run responses of the RER to shocks in the traded sector, as illustrated in fig 1.A and 1.C would seem to lend some support to this observation as well.

In summary, our analysis suggests a tradeoff between alternative modes of adjustment. In economies having less flexible production conditions and less flexible labor markets more of the

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26 We have also graphed the rate of convergence of the RER. It illustrates the same behavior of the real exchange rate, although from a slightly different perspective.
burden of adjustment following a structural change is imposed on the exchange rate, leading to more volatility in the short run and a faster rate of convergence over time.
### Table 1

**Half-Lives and Average Annual Hours Actually Worked per Person in Employment**

<table>
<thead>
<tr>
<th>Country</th>
<th>Half-Life</th>
<th>Average Annual Hours in 1998</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>3.45</td>
<td>1856</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.46</td>
<td>1609</td>
</tr>
<tr>
<td>Canada</td>
<td>6.58</td>
<td>1799</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.16</td>
<td>1511</td>
</tr>
<tr>
<td>Finland</td>
<td>1.95</td>
<td>1761</td>
</tr>
<tr>
<td>France</td>
<td>2.46</td>
<td>1553</td>
</tr>
<tr>
<td>Germany</td>
<td>2.35</td>
<td>1489</td>
</tr>
<tr>
<td>Greece</td>
<td>2.32</td>
<td>1925</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.59</td>
<td>1721</td>
</tr>
<tr>
<td>Italy</td>
<td>2.39</td>
<td>1639</td>
</tr>
<tr>
<td>Japan</td>
<td>3.76</td>
<td>1842</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.22</td>
<td>1366</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2.25</td>
<td>1825</td>
</tr>
<tr>
<td>Norway</td>
<td>1.88</td>
<td>1400</td>
</tr>
<tr>
<td>Portugal</td>
<td>3.85</td>
<td>1747</td>
</tr>
<tr>
<td>Spain</td>
<td>3.12</td>
<td>1834</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.46</td>
<td>1638</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.61</td>
<td>1589</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2.05</td>
<td>1731</td>
</tr>
</tbody>
</table>

**Source:** Half-life: Papell and Murray (2005), Table 4, Column 3; Average Annual Hours, OECD Employment Outlook 2003, Page 322.
Table 2
Base Parameter Values and Key Steady-State Equilibrium Ratios

A. Base Parameter Values
Preference Parameters: $\gamma = -1.5$, $\theta = 0.5$, $\alpha = 0.30$, $\eta = 1$
Foreign Interest Rate: $r = 0.06$
Productivity Parameters: $\phi = 1.5$, $\psi = 1$, Sectoral Adjustment Cost: $h = 30$
Government Expenditures: $G_T = 0.045$, $G_N = 0.12$

B. Key Steady-State Equilibrium Ratio

| Traded Sector More Capital Intensive and Elasticity of Substitution = 1; $m = 0.35$, $n = 0.25$. |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\frac{K_T}{L_T}$ | $\frac{K_N}{L_N}$ | $\frac{K_T}{Y_T}$ | $\frac{K_N}{Y_N}$ | $\frac{K}{Y}$ | $L$ | $\frac{Y_T}{Y}$ | $\frac{G_T}{Y_T}$ | $\frac{G_N}{Y_N}$ |
| 10.83 | 6.70 | 3.14 | 4.17 | 3.74 | 0.299 | 0.41 | 0.116 | 0.399 |

| Nontraded Sector More Capital Intensive and Elasticity of Substitution = 1; $m = 0.25$, $n = 0.35$. |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\frac{K_T}{L_T}$ | $\frac{K_N}{L_N}$ | $\frac{K_T}{Y_T}$ | $\frac{K_N}{Y_N}$ | $\frac{K}{Y}$ | $L$ | $\frac{Y_T}{Y}$ | $\frac{G_T}{Y_T}$ | $\frac{G_N}{Y_N}$ |
| 9.33 | 15.08 | 3.56 | 5.83 | 4.83 | 0.281 | 0.44 | 0.127 | 0.317 |

NB $Y \equiv Y_T + \sigma Y_N$ denotes GDP defined in terms of traded good as numeraire
<table>
<thead>
<tr>
<th>$S_N, S_T$</th>
<th>$\eta \rightarrow$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8, 0.8</td>
<td>$L$</td>
<td>0.439</td>
<td>0.329</td>
<td>0.205</td>
<td>0.426</td>
<td>0.319</td>
<td>0.181</td>
</tr>
<tr>
<td>1, 1</td>
<td>$L$</td>
<td>0.394</td>
<td>0.299</td>
<td>0.176</td>
<td>0.369</td>
<td>0.281</td>
<td>0.164</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$L$</td>
<td>0.364</td>
<td>0.278</td>
<td>0.166</td>
<td>0.326</td>
<td>0.254</td>
<td>0.154</td>
</tr>
</tbody>
</table>
# Table 4
## Short-run and Long-run Real Exchange Rate Elasticities

### A. Elasticity with respect to $G_T$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>$\eta \rightarrow$</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor Supply</td>
<td>$\eta = 0.5$</td>
<td>$\eta = 1$</td>
<td>$\eta = 2$</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\sigma(0)$</td>
<td>-0.025</td>
<td>-0.014</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\sigma(0)$</td>
<td>-0.022</td>
<td>-0.012</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\sigma(0)$</td>
<td>-0.021</td>
<td>-0.010</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### B. Elasticity with respect to $G_N$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>$\eta \rightarrow$</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor Supply</td>
<td>$\eta = 0.5$</td>
<td>$\eta = 1$</td>
<td>$\eta = 2$</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\sigma(0)$</td>
<td>0.116</td>
<td>0.145</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\sigma(0)$</td>
<td>0.111</td>
<td>0.145</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\sigma(0)$</td>
<td>0.107</td>
<td>0.145</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The benchmark parameterization of the utility and the production functions ($s_T = s_N = \eta = 1$) are indicated in bold.
### C. Elasticity with respect to $\phi$

<table>
<thead>
<tr>
<th>Labor Supply</th>
<th>$s_N, s_T$</th>
<th>$\eta \to \eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>1.022 0.969</td>
<td>1.032 1.016</td>
<td>1.065</td>
<td>1.089</td>
<td>1.028</td>
<td>0.949</td>
<td>1.040</td>
</tr>
<tr>
<td>Elastic</td>
<td>1.016 0.947</td>
<td>1.026 1.011</td>
<td>1.054</td>
<td>1.114</td>
<td>1.025</td>
<td>0.908</td>
<td>1.038</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor Supply</th>
<th>$s_N, s_T$</th>
<th>$\eta \to \eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>-0.604 -0.839</td>
<td>-0.624 -0.839</td>
<td>-0.688 -0.839</td>
<td>-0.598 -0.839</td>
</tr>
<tr>
<td>Elastic</td>
<td>-0.386 -0.793</td>
<td>-0.401 -0.793</td>
<td>-0.445 -0.793</td>
<td>-0.358 -0.793</td>
</tr>
</tbody>
</table>

### D. Elasticity with respect to $\psi$

<table>
<thead>
<tr>
<th>Labor Supply</th>
<th>$s_N, s_T$</th>
<th>$\eta \to \eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 1$</th>
<th>$\eta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>-0.211 -0.759</td>
<td>-0.222 -0.759</td>
<td>-0.253 -0.759</td>
<td>-0.139 -0.759</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic</td>
<td>-0.598 -0.839</td>
<td>-0.624 -0.839</td>
<td>-0.688 -0.839</td>
<td>-0.598 -0.839</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The benchmark parameterization of the utility and the production functions ($s_T = s_N = \eta = 1$) are indicated in bold.
Table 5

Speeds of Convergence

A. Increase in $G_T$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>η →</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>η = 0.5</td>
<td>η = 1</td>
<td>η = 2</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\kappa(0)$</td>
<td>0.093</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>0.008</td>
<td>0.020</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\kappa(0)$</td>
<td>0.059</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>-0.003</td>
<td>0.014</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\kappa(0)$</td>
<td>0.046</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>-0.008</td>
<td>0.012</td>
</tr>
</tbody>
</table>

B. Increase in $G_N$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>η →</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>η = 0.5</td>
<td>η = 1</td>
<td>η = 2</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\kappa(0)$</td>
<td>0.081</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>0.152</td>
<td>0.022</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\kappa(0)$</td>
<td>0.054</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>0.108</td>
<td>0.016</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\kappa(0)$</td>
<td>0.042</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\kappa}$</td>
<td>0.089</td>
<td>0.013</td>
</tr>
</tbody>
</table>

The benchmark parameterization of the utility and the production functions ($s_T = s_N = \eta = 1$) are indicated in bold.
### C. Increase in $\phi$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta \rightarrow \eta = 0.5$</td>
<td>$\eta = 1$</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\kappa(0)$ 0.086 0.394</td>
<td>$\tilde{\kappa}$ 0.026 0.020</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\kappa(0)$ 0.056 0.228</td>
<td>$\tilde{\kappa}$ 0.021 0.015</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\kappa(0)$ 0.044 0.167</td>
<td>$\tilde{\kappa}$ 0.019 0.012</td>
</tr>
</tbody>
</table>

### D. Increase in $\psi$

<table>
<thead>
<tr>
<th>$s_N, s_T$</th>
<th>Traded Sector More Capital Intensive</th>
<th>Non-Traded Sector More Capital Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta \rightarrow \eta = 0.5$</td>
<td>$\eta = 1$</td>
</tr>
<tr>
<td>0.8, 0.8</td>
<td>$\kappa(0)$ 0.332 0.328</td>
<td>$\tilde{\kappa}$ 0.024 0.019</td>
</tr>
<tr>
<td>1, 1</td>
<td>$\kappa(0)$ 0.200 0.195</td>
<td>$\tilde{\kappa}$ 0.019 0.014</td>
</tr>
<tr>
<td>1.11, 1.11</td>
<td>$\kappa(0)$ 0.150 0.146</td>
<td>$\tilde{\kappa}$ 0.017 0.011</td>
</tr>
</tbody>
</table>

The benchmark parameterization of the utility and the production functions ($s_T = s_N = \eta = 1$) are indicated in bold.
**Figure 1**
Real Exchange Rate Dynamics under Different Labor Supply Decisions
(Elasticity of Substitution between Capital and Labor = 1 and Elasticity of Substitution between Consumption and Leisure = 1, Solid line---Fixed Labor Supply, Dotted line –Elastic Labor Supply)

A. Increase in Government Expenditure on Traded Goods from 0.045 to 0.06

B. Increase in Government Expenditure on Nontraded Goods from 0.12 to 0.15

C. Productivity of Traded Sector increased from 1.5 to 1.65.

D. Productivity of Nontraded Sector Increase from 1 to 1.1
References


Gavin, M. (1992), “Income Effects of Adjustment to a Terms of Trade Disturbance and the Demand


