Tax Competition with Asymmetric Market Structures:
The Role of Policy Instruments

By

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Abstract

We analyze the location choice of a multinational corporation (MNC) between two host countries with different market structures, i.e., the number of competing domestic firms in them. We consider the effects of import tariff and lump-sum subsidy instruments on the MNC’s choice. Our findings include: (i) when the domestic firms export, with lump-sum subsidy, the country with fewer firms always gets the MNC. In contrast, with tariffs, the country with more firms gets the MNC if they are sufficiently inefficient, (ii) when the domestic firms do not export, the country with more domestic firms may get the MNC when the domestic firms are sufficiently inefficient with either of the two instruments, and (iii) with either instrument, the MNC location decision may crucially depend on which instrument is used to attract the MNC.

Keywords: Foreign direct investment; multinational corporation, tax competition, lump-sum subsidy; import tariff; market structure

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1 Introduction

Over the past few decades, many governments across the globe have been very active in promoting their countries as investment locations for multinational corporations (MNC). Given high fixed costs associated with establishing a physical plant in a foreign country, MNCs typically look for both fundamentals and incentives in the shortlisting of alternative locations, and at the end choose a specific location (see, for example, Oman, 2000; Raff, 2004). Incentives are generally recognized as secondary to more fundamental determinants, such as market size, access to raw materials and availabilities to skilled labor (UNCTAD, 2000). However, the importance of fiscal incentives are now being increasingly recognized. Potential host countries compete with each other by offering fiscal and other incentives to attract the MNC within their borders (Kayalica and Espinosa-Ramirez, 2009; Davies, 2004; Barrols and Cabral, 2000; Raff, 2004). According to a survey of tax incentive regime in over 45 countries from all regions of the world, nearly all countries surveyed offer incentives that target specific sectors (UNCTAD, 2000).

The range of specific fiscal incentives offered by potential host countries is very wide indeed. For example, in the 1990s, Ford built a $700 million automobile assembly plant in Rio Grande do Sul. Ford was given a package of subsidies by the Rio Grande do Sul government which included $250 million in straight subsidies for the construction of infrastructure related to the plant and temporary exemptions from value-added taxes and import duties for plant machinery (Hanson, 2001). Germany gave Dow company $6.8 billion lump-sum subsidy for a plant employing 2000 people, which translates to 3.4 million per job (Moran, 1998). In 1991 Portugal provided a lump sum subsidy and promised tax breaks on future earnings to Ford and Volkswagen for constructing a jointly operated automobile production facility in the country (Hanson, 2001). In 1996, Intel chose Costa Rica over alternative sites in Brazil, Chile, Indonesia and Mexico to build a $300 million semi-conductor assembly and testing plant. The firm was given exemptions from (i) import duties on raw materials,
components and capital goods; (ii) export, sales, excise and municipal taxes; and (iii) taxes on corporate income during the first eight years following an investment, with a 50 per cent exemption applying for the next four years (Hanson, 2001). Canada has high trade barriers for dairy products, and large European companies such as Nestle, Danone, and Parmalat have entered the Canadian dairy product market through Canadian affiliates. Similarly, U.S. trade barriers for foreign wines and dairy products have led European companies to purchase wineries and build dairy plants in the United States (Bolling and Somwaru 2001).

In 2001, Canon Inc. decided to establish a large production facility in East Asia. Vietnam out-compete its rival the Philippines, by offering a substantial incentives package, including a ten-year tax holiday.

There is a small sub-literature on tax competition for foreign direct investment (FDI) that this paper belongs to. Research studying ‘subsidy games’ between governments aiming to attract FDI demonstrates that the equilibrium distribution of FDI between countries with subsidies may well be significantly different from that without subsidies, where each country implements its optimal incentive scheme (Haaparanta, 1996; Motta and Norman, 1996; Barros and Cabral, 2000). The effect of tariffs on the behavior of MNCs is demonstrated by Brainard (1997) and Carr et al. (2001) who predict that when import duties are high, MNCs embark on FDI to circumvent high tariffs: the so-called tariff-jumping FDI.

Haufler and Wooton (1999) focus on the differences in market size between the host countries, while assuming away the existence of domestic producers. They found that the bigger of the two potential host countries will win the competition for FDI, irrespective of whether lump-sum subsides or output subsides are offered. Bjorvatn and Eckel (2006) allow for one domestic firm in one of the two countries in the Haufler and Wooton framework, i.e., the host countries are different in two dimensions: market size and number of domestic firms in them. Guariglia et al. (2006) consider one domestic firm in each potential host country and focus on differences in efficiency levels of the domestic firms.
There is now an empirical and theoretical literature which tries to identify the characteristics of firms that export. Helpman, Melitz and Yeaple (2004), Tomiura (2007) and Yasar and Paul (2007) point out that the most productive firms engage in FDI, the next most productive ones export, and the least productive firms produce only for domestic markets. The self-selection into export markets by the profitable and efficient firms is due to the huge amount of sunk cost associated with export market entry (Clerides et al., 1998, Melitz, 2003, Aw et al., 2007). Greenaway and Kneller (2007) and Breau and Rigby (2006) provide empirical evidence that on average the productivity of exporting firms is higher than that of non-exporting firms.

In this paper, we focus on different market structure in the two potential host countries, i.e., different number of domestic firms in them. We assume, like others in the literature, that the MNC can serve both markets from wherever it locates. As for the domestic firms, following empirical finding on differential access to export markets as mentioned before, we consider two scenarios depending on whether domestic firms export or not. The main focus of the analysis is however on the choice of instruments. In particular, we consider lump-sum subsides and import tariffs as two alternative policy instruments which the host countries use to compete with each other for FDI, and examine if the choice of an instrument can play a crucial role in the determination of the location equilibrium.

The lay out of the paper is as follows. Section 2 describes the basic model. Section 3 then considers the problem of the MNC’s location choice. This section is divided on several subsections depending on whether the domestic firms export or not, and on the nature of instrument used by the host countries to compete with each other. Finally, some concluding remarks are made in section 4.

2 The general model

We consider a partial equilibrium model of a homogeneous good in which a multinational
corporation (MNC, labeled \( M \)) wants to invest in one of two host countries, labeled \( A \) and \( B \). There is a fixed cost \( F \) of setting up a plant in either country. This fixed cost is assumed to be sufficiently large to ensure that the MNC will not choose to operate plants in both countries. For simplicity and without loss of generality, we further assume that there are \( h (> 1) \) number of identical firms in country \( A \), and only one in country \( B \). In order to focus on the effects of tax instrument on MNC’s locational decision between the two hosts with different market structure, we shall assume away any other differences between the two host countries. In particular, we assume that the \( h \) domestic firms in country \( A \) and the one in country \( B \) have the same (constant) marginal and average variable costs \( c \). We assume that the MNC is technologically superior to the domestic firms, and, without loss of generality, the unit variable and marginal costs of the MNC is 0.

The market for the good in the two countries are assumed to be segmented. The inverse demand functions for the good in the two countries are given by

\[
p_i = a - bD_i,
\]

where \( p_i \) and \( D_i \) are respectively the price and the demand for the good in country \( i \) \((i = A, B)\).\(^1\)

The MNC is assumed to serve both markets, but whenever it exports a unit transportation cost \( t \) is incurred, and possibly a unit import tariff \( \tau^j_i \) imposed by the importing country \( i \), where \( \tau^j_i \) is country \( i \)’s unit import tariff in the market of its location, and the superscript \( j \) represents the MNC’s location \((i, j = A, B; j \neq i)\). But for the domestic firms, we shall consider two scenarios: (i) they only serve their respective domestic markets, and (ii) like the MNC they also serve both markets, incurring a unit trade cost for exports. These two cases will be considered below.

\(^1\) Implicit in these inverse demands function are the assumptions of identical preferences and market sizes in the two host countries.
3 Tax competition

We shall now examine the issue of tax competition for winning the MNC. We shall do so under two alternative scenarios about the domestic firms, as mentioned above.

3.1 Domestic firms do not export

We first consider the case where the MNC locates in country $A$. Putting a superscript $A$ to all the variables (indication the location of the MNC), the market-clearing conditions in the two markets are:

$D_A = hq_A + q^*_M,$  $D_B = q_B + q^*_M,$  \hspace{1cm} (2)

where $q_i^A$ is firm $i$’s sales in the market of its location ($i = A, B$), and $q^*_M$ and $q^*_M$ are the MNC’s sales at the host country and exports respectively.

Substituting (2) into (1) yields

$P_A = a - b(hq_A + q^*_M), \quad P_B = a - b(q_B + q^*_M).$  \hspace{1cm} (3)

Each firm’s profit is:

$\pi_A = (P_A - c)q_A, \quad \pi_B = (P_B - c)q_B, \quad \pi_M = P_Aq_M + (P_B - t - \tau_B)q^*_M - F + S_A.$  \hspace{1cm} (4)

Each domestic firm’s profit is simply the difference between revenue and variable costs. For the MNC, total profits consist of profits from the two markets, minus fixed costs $F$, and plus a lump-sum subsidy payment $S_A$ received from the host country’s government.

Assuming Cournot competition, the first-order profit-maximizing conditions are:

$P_A - c = bq_A, \quad P_B - c = bq_B, \quad P_A = bq^*_M, \quad P_B - t - \tau_B = bq^*_M.$  \hspace{1cm} (5)
From (3) and (5), we solve the closed-form solutions as:

\[ P_A = \frac{a + hc}{h + 2}, \quad P_B = \frac{a + c + t + \tau_B^A}{3}, \quad q_A = \frac{a - 2c}{b(h + 2)}, \quad q_M = \frac{a + hc}{b(h + 2)}. \]

\[ q_B^A = \frac{a - 2c + t + \tau_B^A}{3b}, \quad q_M^A = \frac{a + c - 2(t + \tau_B^A)}{3b}. \] (6)

When the MNC locates in country B, the solutions can be analogously derived as:

\[ P_A = \frac{a + hc + t + \tau_A^B}{h + 2}, \quad P_B = \frac{a + c}{3}, \quad q_A = \frac{a - 2c + t + \tau_A^B}{b(h + 2)}, \quad q_M = \frac{a + c}{3b}, \]

\[ q_B^B = \frac{a - 2c}{3b}, \quad q_M^B = \frac{a + hc - (h + 1)(t + \tau_A^B)}{b(h + 2)}. \] (7)

To guarantee positive sales by all firms, we assume

**Assumption 1.** \( \frac{(h+1)t}{h+2} < c < \frac{a}{2} \).

Substituting the first-order profit-maximizing conditions in the expressions for profits and then using (6) and (7), we find the solutions for profits for each firm under the two scenarios as:

\[ \pi_A = b(q_A^2) = \frac{(a - 2c)^2}{b(h + 2)^2}, \quad \pi_B = b(q_B^2) = \frac{(a - 2c + t + \tau_B^A)^2}{b(h + 2)^2}, \]

\[ \pi_A = b(q_B^2) = \frac{(a - 2c + t + \tau_A^B)^2}{9b}, \quad \pi_B = b(q_B^2) = \frac{(a - 2c)^2}{9b}. \] (8)

\[ \pi_M = b(q_M^2) + b(q_M^A)^2 - F + S_A = \frac{(a + hc)^2}{b(h + 2)^2} + \frac{[a + c - 2(t + \tau_B^A)]^2}{9b} - F + S_A, \]

\[ \pi_M = b(q_M^2) + b(q_M^B)^2 - F + S_B = \frac{(a + c)^2}{9b} + \frac{[a + hc - (h + 1)(t + \tau_A^B)]^2}{b(h + 2)^2} - F + S_B. \]

We now consider the situation where the host governments actively try to persuade the MNC locate in their respective countries with the carrots of either lump-sum subsidies.
or unit import tariff. Each host tries to maximize the social welfare while setting up the tax or subsidy strategy. Once they find it is not worthwhile to subsidize beyond a certain level, they will quit the competition.

The utility function implicit behind the demand function is a quasi-linear one of the form \( u(D) = aD - bD^2 / 2 + Z \), where \( D \) and \( Z \) are the domestic consumption of the oligopoly good and the numeraire good respectively. With this utility function, the indirect utility function can easily be derived as \( u = (a - p)^2 / (2b) + I \) where \( I \) is the sum of labor income, domestic firms’ profits and the import tariff revenue, minus any lump-sum payment to the MNC. Substituting for the solution for price and domestic profits obtained in (6), (7) and (8) into this indirect utility function, we derive the levels of social welfare in country \( A \) when the MNC locates in country \( A \) and \( B \) respectively as:

\[
U_A^A = \frac{[(h + 1)a - hc]^2}{2b(h + 2)^2} + \frac{h(a - 2c)^2}{b(h + 2)^2} - S_A, \tag{9}
\]

\[
U_A^B = \frac{[(h + 1)a - hc - t - \tau_A^B]^2}{2b(h + 2)^2} + \frac{h(a - 2c + t + \tau_A^B)^2}{b(h + 2)^2} + \frac{\tau_A^B [a + hc - (h + 1)(t + \tau_A^B)]}{b(h + 2)},
\]

The first term is the consumer surplus, and the second term is the producer surplus in both equations, and the third term in the second equation is country \( A \)’s tariff revenue from imports.

Similarly, we derive the levels of social welfare in country \( B \) when the MNC locates in country \( A \) and \( B \) respectively as:

\[
U_B^A = \frac{(2a - c - t - \tau_A^B)^2}{18b} + \frac{(a - 2c + t + \tau_A^B)^2}{9b} + \frac{\tau_A^B [a + c - 2(t + \tau_A^B)]}{3b},
\]

\[
U_B^B = \frac{(2a - c)^2}{18b} + \frac{(a - 2c)^2}{9b} - S_B, \tag{10}
\]

\footnote{Without any loss of generality, we shall ignore labor income.}
### 3.1.1 The case of lump-sum subsidies

We shall now consider the problem of the MNC’s location choice when the host countries use only lump-sum subsidies to compete with each other, that is

\[ \tau_A^A = \tau_B^B = \tau_A^B = \tau_B^A = 0. \]  

(11)

Substituting (11) into the foreign firm’s profit function (8) we find:

\[
\begin{align*}
\pi_A^M &= \frac{(a + hc)^2}{b(h + 2)^2} + \frac{(a + c - 2t)^2}{9b} - F + S_A, \\
\pi_B^M &= \frac{(a + c)^2}{9b} + \frac{(a + hc - (h + 1)t)^2}{b(h + 2)^2} - F + S_B.
\end{align*}
\]

In the absence of any government policy, the difference between the MNC profits under two alternative location decisions is:

\[
\pi_A^M - \pi_B^M = \frac{-t(h - 1)}{9b(h + 2)^2} [2a(2h + 1) - 2c(7h + 8) + t(5h + 7)].
\]

(12)

Substituting (11) into the (9), we get the utility functions for country A:

\[
\begin{align*}
U_A^A &= \frac{[(h + 1)a - hc]^2}{2b(h + 2)^2} + \frac{h(a - 2c)^2}{b(h + 2)^2} - S_A, \\
U_A^B &= \frac{[(h + 1)a - hc - t]^2}{2b(h + 2)^2} + \frac{h(a - 2c + t)^2}{b(h + 2)^2},
\end{align*}
\]

Equating \( U_A^A = U_A^B \), we solve for the critical subsidy level \( \tilde{S}_A \) at which the government of country A is indifferent between whether the MNC locates in country A or B:

\[
\tilde{S}_A = \frac{-2ta(h - 1) + 6thc - t^2(2h + 1)}{2b(h + 2)^2}.
\]

Clearly, the government of country A would not be willing to offer any subsidy that is higher than the above critical value.
Similarly, for the government in country $B$ the critical subsidy level can be obtained as:

$$\tilde{S}_B = \frac{t(2c - t)}{6b}.$$  

From the above two equations, we find:\footnote{Let $f(h) = -6a - 2c(h - 4) + t(h - 1)$. Clearly, $f(1) = -6(a - c)$ and $f'(h) = -2c + t$. From assumption 1, it follows that $f(1) < 0$ and $f'(h) < 0$ for all $h \geq 1$. Therefore, $f(h) < 0$ for all $h \geq 1$.}

$$\tilde{S}_A - \tilde{S}_B = \frac{t(h - 1)}{6b(h + 2)^2} [-6a - 2c(h - 4) + t(h - 1)] < 0.$$  

That is, the government in country $B$ will always be willing to pay a higher subsidy level than that in country $A$. But will country $B$ win the competition for the MNC? It will if $c$ is sufficiently small so that (12) is negative. Then the MNC would make a bigger profit (not counting any subsidy) by locating itself in country $B$ rather than in country $A$; and, as we find out, country $B$ will be able to seal the deal by offering a higher subsidy. However, when $c$ is sufficiently large so that (12) is positive, it is not clear which country is able to deliver a higher profit (counting subsidy) for the MNC. It can be derived that if $c > \bar{c}$ ($c < \bar{c}$) country $A$ ($B$) will win the competition for the MNC, where

$$\bar{c} = \frac{2a(4h + 11) + t(7h + 17)}{2(11h + 28)} > 0. \quad (13)$$

In fact to attract the MNC, country $A$ (or $B$) does not have to pay subsidy as high as $\tilde{S}_A$ (or $\tilde{S}_B$), as in Haufler and Wooton (2000), Kind et al. (2000) and Bjorvatn and Eckel (2006). By offering a subsidy $\tilde{S}_A$ or $\tilde{S}_B$, country $A$ (or $B$) is able to appropriate the entire location rent without inducing relocation, where $\tilde{S}_A = \tilde{S}_B + \pi^A_M - \pi^B_M$ (or $\tilde{S}_B = \tilde{S}_A + \pi^A_M - \pi^B_M$). The above results are stated formally in the following proposition.

**Proposition 1.** Consider the case where the governments of the host countries use lump-sum subsidies to compete for FDI and domestic firms do not export. The country with a fewer (larger) number of domestic firm will win the competition for the MNC if $c < \bar{c}$ ($c > \bar{c}$).
First of all, note that in the absence of any transportation cost, neither country would offer any subsidy and the MNC would be indifferent between locating in the two countries. In the presence of transportation cost, each country can increase its consumers’ surplus by persuading the MNC to locate there. As for profits of the MNC, note that it has two sources: profits from the local market and profits from exports. Profits from the former is higher if it locates in the country with fewer firms. But profits from exports is higher if it locates in the country with more domestic firms. In fact, if the domestic firms are sufficiently inefficient (so that competition from domestic firms are less important), a higher profit from exports can outweigh lower profits from local sales if the MNC locates in the the country with more domestic firms.

3.1.2 The case of import tariffs

In this section we consider unit import tariff as an instrument for the host government to compete for the MNC, and assume that there are no lump-sum taxes,

\[ S_A = S_B = 0. \]  

(14)

Substituting (14) into each host’s utility function (9), we derive this under two alternatives for its locations:

\[ U_A^A = \frac{[(h + 1)a - hc]^2}{2b(h + 2)^2} + \frac{h(a - 2c)^2}{b(h + 2)^2}, \]

\[ U_A^B = \frac{[(h + 1)a - hc - t - \tau_A^B]^2}{2b(h + 2)^2} + \frac{h(a - 2c + t + \tau_A^B)^2}{b(h + 2)^2} + \frac{\tau_A^B [a + hc - (h + 1)(t + \tau_A^B)]}{b(h + 2)}. \]

The first term is the consumer surplus, and the second term is the producer surplus in both equations, and the third term is the tariff revenue for the importing country.

The importing country, country \( A \) maximizes its social welfare by choosing the unit import tariff rate optimally. By equating the partial derivative of \( U_A^B \) with respect to \( \tau_A^B \) to
we get
\[ \tau_A^B = \frac{(2h + 1)a + (h^2 - h)c - (h^2 + h + 1)t}{2h^2 + 4h + 3}. \]  

(15)

Similarly, the optimal level of unit import tariff by country B if the MNC locates in country A is,
\[ \tau_B^A = \frac{a - t}{3}. \]  

(16)

Substituting (15) and (16) into (8), we find the difference in the MNC’s equilibrium profits from the two alternative location choices
\[ \pi_M^A - \pi_M^B = \frac{-(h - 1)M}{81b(h + 2)^2(2h^2 + 4h + 3)^2}, \]
where the expression for \( M \) is in appendix 1. It is also shown in appendix 1 that there is a critical value \( \tilde{c} > 0 \) such that
\[ c \gtrless \tilde{c} \iff M \lesssim 0 \iff \pi_M^A \gtrless \pi_M^B. \]

From the above, the following proposition follows.

**Proposition 2.** When the host governments implement unit import tariff and the domestic firms do not export, the MNC would locate in the country with more domestic firms if \( c > \tilde{c} \). Otherwise, it would locate in the country with fewer domestic firms.

The intuition is similar to the one for proposition 1.

### 3.1.3 Comparing the two instruments

From propositions 1 and 2 we know that the country with more domestic firms will win the competition for the MNC if the domestic firms are sufficiently inefficient. However, the critical values of ‘efficiency’ of the domestic firms are different for the two tax instruments.
Comparing \( \bar{c} \) (which is the critical value with import tariffs) with \( \bar{c} \) (critical value with lump-sum subsidy), we find that the level of unit transportation cost, \( t \), is an important parameter. In particular,

\[
\bar{c} \geq \bar{c} \iff t \geq \bar{t},
\]

where

\[
\bar{t} = \frac{S + 6\sqrt{V}}{R} > 0,
\]

the parameters \( S, V \) and \( R \) are defined in appendix 2.

In view of propositions 1 and 2 and the above result, we derive the following results.

**Proposition 3.** When the domestic firms do not export, we have:

1. If \( 0 < t < \bar{t} \) so that \( 0 < \bar{c} < \bar{c} \), then

   (a) when \( 0 < c < \bar{c} \), the country with fewer domestic firms get the MNC, either with tariff or with lump-sum subsidy;

   (b) when \( \bar{c} < c < \bar{c} \), the country with more domestic firms gets the MNC if tariff is used as an instrument, but the country with fewer domestic firms gets the MNC with lump-sum subsidy;

   (c) when \( c > \bar{c} \), the country with more domestic firms gets the MNC, either with tariff or lump-sum subsidy.

2. If \( t > \bar{t} > 0 \) so that \( \bar{c} > \bar{c} > 0 \), then

   (a) when \( 0 < c < \bar{c} \), the country with fewer domestic firms gets the MNC with either tariff or lump-sum subsidy;

   (b) when \( \bar{c} < c < \bar{c} \), the country with more domestic firms gets the MNC with lump-sum subsidy, but the country with fewer domestic firms gets the MNC with tariffs.
(c) when $c > \hat{c}$, the country with more domestic firms gets the MNC, either with tariff or lump-sum subsidy.

From results 1(b) and 2(b) above, we note that the outcome of the tax competition can crucially depend on which instrument is used by the governments to compete for the MNC.

### 3.2 Domestic firms export

In this section we assume that domestic firms can also export. This model is essentially the reciprocal dumping model of Brander & Krugman (1983). In this case, the market-clearing conditions are:

$$D_A^A = hq_A^A + q_M^A + q_B^A, \quad D_B^A = q_B^A + q_M^A + hq_A^A.$$  

Substituting the above equations into demand function (1), we get:

$$p_A^A = a - b \left( hq_A^A + q_M^A + q_B^A \right), \quad p_B^A = a - b \left( q_B^A + q_M^A + hq_A^A \right). \quad (17)$$

Profit of each firm is

$$\pi_A^A = (P_A^A - c) q_A^A + (P_B^A - c - t - \tau_B^A) q_B^A, \quad \pi_B^A = (P_B^A - c) q_B^A + (P_A^A - c - t - \tau_B^A) q_A^A,$$

$$\pi_M^A = P_A^A q_M^A + (P_B^A - t - \tau_B^A) q_M^A - F + S_A. \quad (18)$$

Assuming Cournot behavior, the first-order profit-maximizing conditions are:

$$P_A^A - c = bq_A^A, \quad P_B^A - c - t - \tau_B^A = bq_A^A, \quad P_A^A - c = bq_B^A,$$

$$P_A^A - c - t - \tau_B^A = bq_B^A, \quad P_A^A = bq_M^A, \quad P_B^A - t - \tau_B^A = bq_M^A.$$
The closed-form solutions from the above equations are:

\[
\begin{align*}
P_A &= \frac{a + (h+1)c + t + \tau_A^A}{h+3}, \quad P_B = \frac{a + (h+1)c + (h+1)(t + \tau_A^A)}{h+3}, \\
q_A &= \frac{a - 2c + t + \tau_A^A}{b(h+3)}, \quad q_A^* = \frac{a - 2c - 2(t + \tau_B^A)}{b(h+3)}, \\
q_B &= \frac{a - 2c + (h+1)(t + \tau_B^A)}{b(h+3)}, \quad q_B^* = \frac{a - 2c - (h+2)(t + \tau_A^A)}{b(h+3)}, \\
q_M &= \frac{a + (h+1)c + t + \tau_A^A}{b(h+3)}, \quad q_M^* = \frac{a + (h+1)c - 2(t + \tau_B^A)}{b(h+3)},
\end{align*}
\]  

(19)

When the MNC locates in country B, the closed-form solutions are analogously derived as:

\[
\begin{align*}
P_B &= \frac{a + (h+1)c + h(t + \tau_B^B)}{h+3}, \quad P_A = \frac{a + (h+1)c + 2(t + \tau_B^B)}{h+3}, \\
q_A &= \frac{a - 2c + 2(t + \tau_A^B)}{b(h+3)}, \quad q_A^* = \frac{a - 2c - 3(t + \tau_B^B)}{b(h+3)}, \\
q_B &= \frac{a - 2c + h(t + \tau_B^B)}{b(h+3)}, \quad q_B^* = \frac{a - 2c - (h+1)(t + \tau_A^B)}{b(h+3)}, \\
q_M &= \frac{a + (h+1)c + h(t + \tau_B^B)}{b(h+3)}, \quad q_M^* = \frac{a + (h+1)c - (h+1)(t + \tau_A^B)}{b(h+3)}.
\end{align*}
\]  

(20)

Once again, the expressions of profits of each firm under the two scenarios are:

\[
\begin{align*}
\pi_A &= b(q_A^2 + b(q_A^*)^2) = \frac{(a - 2c + t + \tau_A^A)^2}{b(h+3)^2} + \frac{[a - 2c - 2(t + \tau_B^A)]^2}{b(h+3)^2}, \\
\pi_B &= b(q_B^2 + b(q_B^*)^2) = \frac{[a - 2c + 2(t + \tau_B^A)]^2}{b(h+3)^2} + \frac{[a - 2c - 3(t + \tau_B^B)]^2}{b(h+3)^2}, \\
\pi_M &= b(q_M^2 + b(q_M^*)^2) = \frac{[a - 2c + (h+1)(t + \tau_B^A)]^2}{b(h+3)^2} + \frac{[a - 2c - (h+2)(t + \tau_A^A)]^2}{b(h+3)^2},
\end{align*}
\]
\( \pi_B^B = b(q_B^B)^2 + b(q_B^A)^2 = \frac{[a - 2c + h(t + \tau_B)]^2}{b(h + 3)^2} + \frac{[a - 2c - (h + 1)(t + \tau_B)]^2}{b(h + 3)^2}, \) \hspace{1cm} (21)

\[
\begin{align*}
\pi_A &= b(q_A^A)^2 + b(q_A^*)^2 - F + S_A \\
&= \frac{[a + (h + 1)c + t + \tau_A^A]^2}{b(h + 3)^2} + \frac{[a + (h + 1)c - 2(t + \tau_B^A)]^2}{b(h + 3)^2} - F + S_A, \\
\pi_M &= b(q_M^B)^2 + b(q_M^*)^2 - F + S_B \\
&= \frac{[a + (h + 1)c + h(t + \tau_B)]^2}{b(h + 3)^2} + \frac{[a + (h + 1)c - (h + 1)(t + \tau_B)]^2}{b(h + 3)^2} - F + S_B.
\end{align*}
\]

When all firms (including the domestic firms) export, the levels of social welfare in country \( A \) is calculated as before (in the case of no domestic exports); the only difference is that producer surplus from exports is added. Under the two alternative location choices for the MNC, the expressions for the welfare levels are:

\[
\begin{align*}
U_A &= \frac{[(h + 2)a - (h + 1)c - t - \tau_A^A]^2}{2b(h + 3)^2} + \frac{h(a - 2c + t + \tau_A^A)^2}{b(h + 3)^2} + \frac{h[a - 2c - 2(t + \tau_B^A)]^2}{b(h + 3)^2} \\
&+ \frac{\tau_A^A[a - 2c - (h + 2)(t + \tau_A^A)]}{b(h + 3)} - S_A, \\
U_B &= \frac{[(h + 2)a - (h + 1)c - 2(t + \tau_B^A)]^2}{2b(h + 3)^2} + \frac{h[a - 2c + 2(t + \tau_B^A)]^2}{b(h + 3)^2} \\
&+ \frac{h[a - 2c - 3(t + \tau_B^A)]^2}{b(h + 3)^2} + \frac{\tau_A^B[2a + (h - 1)c - 2(h + 1)(t + \tau_B^A)]}{b(h + 3)}.
\end{align*}
\]

The first term is the consumer surplus, the second and the third terms are the producer surplus in the domestic market and the foreign market, respectively, and the fourth term is the import tariff revenue in both equations.

Similarly, we derive the levels of social welfare in country \( B \) when the MNC locates
in country $A$ and $B$ respectively as:

$$U_B^A = \frac{[(h + 2) a - (h + 1) c - (h + 1) (t + \tau_B^A)]^2}{2b(h+3)^2} + \frac{[a - 2c + (h + 1) (t + \tau_B^A)]^2}{b(h+3)^2}$$

$$+ \frac{[a - 2c - (h + 2) (t + \tau_B^A)]^2}{b(h+3)^2} + \frac{\tau_B^A (a + c - 2t + ah - ch - 2ht - 2\tau_B^A - 2h\tau_B^A)}{b(h+3)}$$

$$U_B^B = \frac{[(h + 2) a - (h + 1) c - h (t + \tau_B^B)]^2}{2b(h+3)^2} + \frac{[a - 2c + h (t + \tau_B^B)]^2}{b(h+3)^2}$$

$$+ \frac{[a - 2c - (h + 1) (t + \tau_B^B)]^2}{b(h+3)^2} + \frac{\tau_B^B h [a - 2c - 3 (t + \tau_B^B)]}{b(h+3)} - S_B.$$  

As in the case when domestic firms do not export, we shall now consider the question of tax competition under two alternative instruments. These are taken up in turn.

3.2.1 The case of lump-sum subsidy

Focusing on lump-sum subsidies first, and ignoring tariffs ($\tau_A^A = \tau_B^B = \tau_A^B = \tau_B^A = 0$), from (21) we can find the MNC’s profits under alternative locational choice as:

$$\pi_M^A = \frac{[a + (h + 1) c + t]^2}{b(h+3)^2} + \frac{[a + (h + 1) c - 2t]^2}{b(h+3)^2} - F + S_A,$$

$$\pi_M^B = \frac{[a + (h + 1) c + ht]^2}{b(h+3)^2} + \frac{[a + (h + 1) c - (h + 1) t]^2}{b(h+3)^2} - F + S_B. \quad (23)$$

In the absence of subsidies, the difference between $\pi_M^A$ and $\pi_M^B$ is computed from (23) as:

$$\pi_M^A - \pi_M^B = \frac{-2t^2(h - 1)(h + 2)}{b(h+3)^2} < 0. \quad (24)$$
Substituting the solutions into (22) we get:

\[
U_A^A = \frac{[(h+2)a-(h+1)c-t]^2}{2b(h+3)^2} + \frac{h(a-2c+t)^2}{b(h+3)^2} + \frac{h(a-2c-2t)^2}{b(h+3)^2} - S_A,
\]

\[
U_A^B = \frac{[(h+2)a-(h+1)c-2t]^2}{2b(h+3)^2} + \frac{h(a-2c+2t)^2}{b(h+3)^2} + \frac{h(a-2c-3t)^2}{b(h+3)^2}.
\]

In both utility functions, the first term is consumer surplus, the second term is the producer surplus from the domestic market, and the third term is the producer surplus from exports.

Equating \(U_A^A\) and \(U_A^B\), we calculate the maximum level of the subsidy the government in country A will be willing to offer, \(\tilde{S}_A\):

\[
\tilde{S}_A = \frac{2t(h+2)a-2t(h+1)c-t^2(16h+3)}{2b(h+3)^2}.
\]

Similarly, for country B, the maximum subsidy level is:

\[
\tilde{S}_B = \frac{2t(h+2)a-2t(h+1)c-t^2(10h+9)}{2b(h+3)^2}.
\]

From the above two equations we find:

\[
\tilde{S}_A - \tilde{S}_B = \frac{-3t^2(h-1)}{b(h+3)^2} < 0. \quad (25)
\]

Country B is always able to out-compete country A in attracting the MNC. Since the MNC also makes a higher profit (without any subsidy) by locating in country B (see (24)), country B always wins the competition for the MNC. Formally,

**Proposition 4.** Suppose that all firms can export. With lump-sum subsidy as the instrument, the MNC will always locate in the country with fewer domestic firms.
Once domestic firms participate in the international trade through export, all firms compete in both markets and thus the two markets are very interconnected. Note that in the absence of any transportation costs, neither country would offer any subsidy and the MNC would be indifferent between the two countries. This is because location of the MNC gives no party any advantage and this situation is the same when domestic firms do not export. In the presence of some friction in exports (i.e., positive transportation costs), the MNC, in the absence of any subsidy, would unambiguously prefer to locate in the country with fewer domestic firms. This in contrast to the case of no exports by domestic firms, where the MNC would have made more profits by locating in the country with a larger number of domestic firms if the domestic firms were sufficiently inefficient. This is because competition from domestic firms is less of an issue in the present case.

When all firms participate in both markets, two markets are very interconnected. From the perspective of social welfare, the domestic producers’ surplus becomes less important as none of them is able to maintain its market power in its own countries, but consumers’ surplus becomes more important. With a relatively more competitive domestic market in country A, the foreign entry by the MNC would not increase country A’s consumers’ surplus as much as it would for country B’s consumers’ surplus. Thus, country B is willing to offer the MNC more generous subsidy than country A.

The model of the present subsection (the case of lump-sum subsidies and exports by domestic firms) has some common elements with that in Bjorvatn and Eckel (2006) who consider one domestic firm in one host country and none in the other. The country with the domestic firm is also larger than the other country in their framework. They find that the larger country is more likely to win competition for the MNC. In contrast, by focusing on differences in the number of domestic firms in the two host countries with at least one domestic firm in each country, we obtain a very different result, viz., the country with fewer domestic firms always wins the competition.
3.2.2 The case of import tariffs

In this subsection we assume away lump-sum subsidies ($S_A = S_B = 0$) and consider import tariffs as the only instrument for competing for FDI.

With all firms able to export, the levels of social welfare in country $A$ under two alternative location choices of the MNC are obtained from (22) as:

$$U^A_A = \left(\frac{(h+2)a-(h+1)c-t}{2b(h+3)^2}\right)^2 + \frac{h(a-2c+t+\tau_A^A)^2}{b(h+3)^2}$$

$$+ \frac{h\left[a-2c-2\left(t+\tau_A^A\right)\right]^2}{b(h+3)^2} + \frac{\tau_A^A\left[a-2c-(h+2)\left(t+\tau_A^A\right)\right]}{b(h+3)},$$

$$U^B_A = \left(\frac{(h+2)a-(h+1)c-2\left(t+\tau_A^B\right)}{2b(h+3)^2}\right)^2 + \frac{h\left[a-2c+2\left(t+\tau_A^B\right)\right]^2}{b(h+3)^2}$$

$$+ \frac{h\left[a-2c-3\left(t+\tau_B^A\right)\right]^2}{b(h+3)^2} + \frac{\tau_B^A\left[2a+(h-1)c-2(h+1)\left(t+\tau_B^A\right)\right]}{b(h+3)}.$$ (26)

From the above, the optimal tariff levels set by country $A$ (when the MNC locates in country $a$ and $B$ respectively) are computed as:

$$\tau_A^A = \frac{(2h+1)a-(5h+5)c-(h^2+3h+5)t}{(2h^2+8h+11)},$$

$$\tau_A^B = \frac{2a-c(1-h^2)-2t(1+h^2)+4h(a-c)}{4h^2+8+8h}.$$

Similarly, we obtain the optimal tariffs set by country $B$ are:

$$\tau_B^A = \frac{3a-3t+3ah-4ch-2ht+h^2t}{h^2+10h+9},$$

$$\tau_B^B = \frac{3a-9c-9t-ch}{3h+18},$$

Substituting the expressions for optimal tariffs into (21), we find the difference in
MNC’s profits from alternative location choice as:

\[ \pi^A_M - \pi^B_M = b (q^A_M)^2 + b (q^*_A)^2 - b (q^B_M)^2 - b (q^*_B)^2 \]

\[ = \frac{[a + (h + 1) c + t + \tau^A_A]^2}{b (h + 3)^2} + \frac{[a + (h + 1) c - 2 (t + \tau^A_A)]^2}{b (h + 3)^2} \]

\[ - \frac{[a + (h + 1) c + h (t + \tau^B_B)]^2}{b (h + 3)^2} + \frac{[a + (h + 1) c - (h + 1) (t + \tau^B_B)]^2}{b (h + 3)^2} \]

\[ = - \frac{(h - 1) N}{b (2h^2 + 8h + 11)^2 (h^2 + 10h + 9)^2 (3h + 18)^2 (4h^2 + 8h + 8)^2}, \]

where the expression for \( N \) is given in the appendix 3.

It follows from the above equation and the expression for \( N \) that

\[ \pi^A_M - \pi^B_M \geq \pi^*_M \iff N \leq 0 \iff c \geq \hat{c}, \]

where the expression for \( \hat{c} (> 0) \) is given in appendix 3.

From the above, we derive the following proposition:

**Proposition 5.** Suppose that the host governments use import tariffs to compete for FDI and that the domestic firms export. The MNC will locate in the country with more (less) domestic firms if \( c > \hat{c} \) \((c < \hat{c})\).

The result is qualitatively similar to the case where domestic firms do not export (proposition 2). This was not the case when the instrument is a lump-sum subsidy. This is because whereas lump-sum subsidies are non-distortionary and cannot affect import levels, tariffs do restrict the levels of imports. With optimal import tariffs the governments can essentially restrict imports from the domestic firms in the other country to a larger extent than those from the MNC which is more efficient than domestic firms.

### 3.2.3 Comparing the two instruments

From proposition 4 we know that the country with fewer domestic firms will get the MNC if lump-sum subsidy instrument is used, and in proposition 5 we found that the country with
more domestic firms wins the competition for the MNC if the domestic firms are sufficiently inefficient and the policy instrument used is import tariffs. Stately formerly, we have the following proposition:

**Proposition 6.** Consider the case where domestic firms also export. With lump-sum subsidy, the country with fewer domestic firms always gets the MNC. However, with unit import tariffs as an instrument, the country with more domestic firms wins the competition for the MNC if $c > \hat{c}$.

That is, as in the case where domestic firms do not export (proposition 3), the outcome of the tax competition may crucially depend on which policy instrument is used by the host governments.

## 4 Conclusion

Due to high fixed costs associated with foreign direct investments (FDI), multinational corporations (MNCs) often have to choose between alternative locations. With an increasing number of governments competing fiercely to attract FDI, the role of fiscal incentives have become a phenomenon of great importance. Can the location equilibrium depend on what instrument the hosts use to compete for FDI? This question is at the heart of the present paper. In particular, we consider two alternative tax instruments adopted by the host countries to compete for FDI: a lump-sum subsidy and an import tariff.

There are of course many factors that are important determinants of an MNC’s locational choice. In this paper we focus on differences between market structures in two alternative host countries, i.e., the number of existing domestic firms in the two countries are different. To focus on this, we rule out differences in efficiency levels of the domestic firms and other differences such as market size in the host countries.
There is considerable empirical evidence to suggest that not all firms in an industry take part in exports and that MNCs are more likely to export than domestic firms. Because of this, we consider two scenarios depending on whether the domestic firms take part in international trade or not.

In our model, as mentioned above, there are existing domestic firms in each host country. Although FDI benefits the country (that wins the competition for it) through enhanced competition, foreign entry may shift profit away from local firms to the MNC. Hence, intensified market competition is not necessarily in the interest of local governments. We examine the role of two tax instruments, one at a time, on an MNC’s location decision between two host countries with different market structures. We develop a two-country partial equilibrium model for an oligopolistic industry, the market for which is internationally segmented, and firms from one country can potentially export to the other after incurring some trade/transportation costs.

Some of our interesting findings are as following. When the domestic firms in the two host countries do not export, if the domestic firms are not very inefficient, the country with less domestic firms is more likely to win the MNC with either of the two instruments. In the case where domestic firms export, the country with fewer domestic firms always gets FDI with lump-sum subsidy, but the import tariff as an instrument can tilt the FDI location choice in favor of the country which has a larger number of domestic firms. We also find that, with either instrument, it is possible that choice of instrument can be a crucial factor determining where the MNC locates.

Though the model simplifies many aspects of the real world and conducts the analysis with specific functional forms of demand and cost functions. However, the results are suggestive and further research needs to be carried out to test the robustness of the results.
Appendix 1

\[ M = 360at - 432ac - 864ct - 1854ach + 1248aht - 2736cht + 117a^2 + 252t^2 \]
\[ + 681a^2h + 744ht^2h^2 - 1350ach^3 - 216ach^4 + 48ach^5 + 1618ah^2t + 990ah^3t \]
\[ - 2514ac + 288ah^4t - 3408ch^2t + 32ah^5t - 2052ch^3t - 594ch^4t - 66ch^5t \]
\[ + 1132a^2h^2 + 828a^2h^3 + 288a^2h^4 - 567c^2h^2 + 32a^2h^5 - 1215c^2h^3 \]
\[ - 891c^2h^4 - 243c^2h^5 + 895h^2t^2 + 531h^3t^2 + 153h^4t^2 + 17h^5t^2 \]

It can be easily shown that

\[ M \leq 0 \iff c \geq \tilde{c} = \frac{F - 3\sqrt{G}}{E}, \]

where

\[ E = -567h^2 - 1215h^3 - 891h^4 - 243h^5 \]
\[ F = 216a + 432t + 927ah + 1368ht + 1257ah^2 + 675ah^3 + 108ah^4 \]
\[ - 24ah^5 + 1704h^2t + 1026h^3t + 297h^4t + 33h^5t \]
\[ G = 20736at + 154656aht + 5184a^2 + 20736t^2 + 44496a^2h + 131328ht^2 \]
\[ + 506952ah^2t + 974424ah^3t + 1223218ah^4t + 1049562ah^5t + 622932ah^6t \]
\[ + 251896ah^7t + 66168ah^8t + 10152ah^9t + 688ah^{10}t + 163188a^2h^2 \]
\[ + 350040a^2h^3 + 494629a^2h^4 + 485208a^2h^5 + 336228a^2h^6 + 162928a^2h^7 \]
\[ + 52884a^2h^8 + 10368a^2h^9 + 928a^2h^{10} + 387396h^2t^2 + 697404h^3t^2 \]
\[ + 844813h^4t^2 + 716706h^5t^2 + 429477h^6t^2 + 178672h^7t^2 + 49104h^8t^2 \]
\[ + 7992h^9t^2 + 580h^{10}t^2 \]

Appendix 2
\[ R = 312384h + 875207h^2 + 1240865h^3 + 1015232h^4 + 488356h^5 + 127585h^6 + 13843h^7 + 32256 \]
\[ S = -173376a - 735576ah - 1440724ah^2 - 1612276ah^3 - 1090000ah^4 - 434606ah^5 - 92522ah^6 - 7976ah^7 \]
\[ V = 686859264a^2 + 5415088896a^2h + 22704662016a^2h^2 + 63795439920a^2h^3 + 128698399396a^2h^4 + 192096187736a^2h^5 + 215705699289a^2h^6 + 183784845750a^2h^7 + 118941264201a^2h^8 + 58054669416a^2h^9 + 21003094776a^2h^{10} + 5454132072a^2h^{11} + 959927028a^2h^{12} + 102395744a^2h^{13} + 4991008a^2h^{14} \]

Appendix 3
\[ N = 32205600at - 41341104ct - 239964768ach + 195428160aht \]
\[-80108352cht - 5900688a^2 - 21861252c^2 + 34161264t^2 - 21206016a^2h \]
\[-130581072c^2h + 202094784ht^2 - 634067100ach^2 - 1000629204ach^3 \]
\[-1043381544ach^4 - 752780616ach^5 - 382487436ach^6 - 136430052ach^7 \]
\[-33407760ach^8 - 5390928ach^9 - 540528ach^{10} - 30672ach^{11} - 768ach^{12} \]
\[+537816168ah^2t + 886838328ah^3t + 974932848ah^4t - 247509108ch^2t \]
\[+752012784ah^5t - 397188756ch^3t + 416582280ah^6t - 378762132ch^4t \]
\[+166332888ah^7t - 216561108ch^5t + 47188512ah^8t - 61175388ch^6t + 9179424ah^9t \]
\[+6742404ch^7t + 1143648ah^{10}t + 13676388ch^8t + 79776ah^{11}t + 5852820ch^9t \]
\[+2304ah^{12}t + 1346688ch^{10}t + 177216ch^{11}t + 12240ch^{12}t + 336ch^{13}t - 2620852a^2h^2 \]
\[-3376404a^2h^3 + 27482004a^2h^4 - 366132969c^2h^2 + 37275156a^2h^5 - 657138705c^2h^3 \]
\[+25781616a^2h^6 - 829824529c^2h^4 + 10952208a^2h^7 - 762342289c^2h^5 + 2904336a^2h^8 \]
\[-513605803c^2h^6 + 455184a^2h^9 - 252674787c^2h^7 + 36864a^2h^{10} - 89434987c^2h^8 \]
\[+1152a^2h^{11} - 22161359c^2h^9 - 3681312c^2h^{10} - 384736c^2h^{11} - 22668c^2h^{12} \]
\[-572c^2h^{13} + 549205308h^2t^2 + 909850140h^3t^2 + 1026216540h^4t^2 + 832683420h^5t^2 \]
\[+500321124h^6t^2 + 225318276h^7t^2 + 75904020h^8t^2 + 18804420h^9t^2 + 3300336h^{10}t^2 \]
\[+382320h^{11}t^2 + 25488h^{12}t^2 + 720h^{13}t^2 \]

It can be easily shown that

\[ N \leq 0 \iff c \leq \hat{c} = \frac{K - 12\sqrt{L}}{J}, \]
where

\[
J = -130581072h - 366132969h^2 - 657138705h^3 - 829824529h^4 \\
-762342289h^5 - 513605803h^6 - 252674787h^7 - 89434987h^8 - 22161359h^9 \\
-3681312h^{10} - 384736h^{11} - 22668h^{12} - 572h^{13} - 21861252
\]

\[
K = 20670552a + 4337064t + 119982384ah + 40054176ht + 317033550ah^2 \\
+500314602ah^3 + 521690772ah^4 + 376390308ah^5 + 191243718ah^6 + 68215026ah^7 \\
+16703880ah^8 + 2695464ah^9 + 270264ah^{10} + 15336ah^{11} + 384ah^{12} \\
+123754554h^2t + 198594378h^3t + 189381066h^4t + 108280554h^5t + 30587694h^6t \\
-3371202h^7t - 6838194h^8t - 2926410h^9t - 673344h^{10}t - 88608h^{11}t \\
-6120h^{12}t - 168h^{13}t
\]
\[ L = 6134401050624 \text{at} + 77599808599104aht + 2071356198912a^2 \\
+ 531679253296t^2 + 25875666603072a^2h + 64071489753216ht^2 \\
+ 46212372144560ah^2t + 173594387808532ah^3 + 463705795342768ah^4t \\
+ 9398079919311696ah^5t + 1504052401834932ah^6t + 1951463332871396ah^7t \\
+ 20900877049320900ah^8t + 18710477830168624ah^9t + 14118177937740308ah^{10}t \\
+ 9027032127117848ah^{11}t + 4904073880339036ah^{12}t + 2264586296530624ah^{13}t \\
+ 887294296200872ah^{14}t + 293808137941780ah^{15}t + 81700117922860ah^{16}t \\
+ 18907346012496ah^{17}t + 3597439583652ah^{18}t + 553711068288ah^{19}t \\
+ 6322314640ah^{20}t + 436810176ah^{21}t + 20849920ah^{22}t + 611152ah^{23}t \\
+ 8256ah^{25}t + 152777233076112a^2h^2 + 566828050527648a^2h^3 \\
+ 29293998933384a^2h^5 + 4525430595482476a^2h^6 + 562181174296180a^2h^7 \\
+ 5718228966712293a^2h^8 + 4821506660438688a^2h^9 + 3397744346089824a^2h^{10} \\
+ 2010947720361328a^2h^{11} + 1001641604276286a^2h^{12} + 419681190082312a^2h^{13} \\
+ 147482502132592a^2h^{14} + 43229121107444a^2h^{15} + 10480850370437a^2h^{16} \\
+ 331391187180a^2h^{18} + 41651239080a^2h^{19} + 4007014124a^2h^{20} + 283075168a^2h^{21} \\
+ 372093292002672ah^{22}t^2 + 13763680a^2h^{22} + 1386700969161456ah^{3}t^2 + 409568a^2h^{23} \\
+ 3724617278274732h^{4}t^2 + 5600a^2h^{24} + 7675334048152080h^5t^2 \\
+ 16944905205450248h^7t^2 + 18951110984877711h^8t^2 + 17858118490028618h^9t^2 \\
+ 14299169674274604h^{10}t^2 + 9782758375481702h^{11}t^2 + 5736537521117643h^{12}t^2 \\
+ 2886235741649620h^{13}t^2 + 1244856525469852h^{14}t^2 + 458978039702828h^{15}t^2 \\
+ 143965982013641h^{16}t^2 + 38148503897874h^{17}t^2 + 8458546916876h^{18}t^2 \\
+ 230727672477h^{20}t^2 + 27316382624h^{21}t^2 + 2496682312h^{22}t^2 + 168900856h^{23}t^2 \\
+ 7921428h^{24}t^2 + 228864h^{25}t^2 + 3056h^{26}t^2 + 2077461178128a^2h^{17} \\
+ 1549639690014h^{19}t^2 + 67476162580ah^{20}t + 12610468158717700h^6t^2 \\
+ 1485206671610196a^2h^4 \]
\( \hat{c} \) is always positive since \( K^2 - 144L = -36(h + 1)^2WY < 0 \), where

\[
W = 130581,072h + 366,132,969h^2 + 657,138,705h^3 + 829824529h^4 + 762342289h^5 \\
+ 513,605,803h^6 + 252674787h^7 + 89434987h^8 + 22161359h^9 + 3681312h^{10} \\
+ 384736h^{11} + 22668h^{12} + 572h^{13} + 21861252,
\]

\[
Y = 894600at + 3639,360aht - 163,908a^2 + 948,924t^2 - 261,240a^2h + 3715896ht^2 \\
+ 6766018ah^2t + 7463002ah^3t + 5389446ah^4t + 2647350ah^5t + 887584ah^6t \\
+ 197,840ah^7t + 27528ah^8t + 2088ah^9t + 64ah^{10}t - 41469a^2h^2 + 250389a^2h^3 \\
+ 304080a^2h^4 + 176872a^2h^5 + 58332a^2h^6 + 10692a^2h^7 + 960a^2h^8 + 32a^2h^9 \\
+ 6874987h^2t^2 + 7807745h^3t^2 + 6015538h^4t^2 + 3291274h^5t^2 + 1299723h^6t^2 \\
+ 368121h^7t^2 + 72480h^8t^2 + 9264h^9t^2 + 668h^{10}t^2 + 20h^{11}t^2
\]
References


