Integrated Reforms of Indirect Taxes in the Presence of Pollution

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Abstract

The literature on indirect tax reforms in pollution-ridden economies is quite limited. This paper, using a model of a small open economy with production and consumption generated pollution, considers the welfare implications of tax reforms within an integrated structure of consumption and production taxes. Specifically, both in the presence and absence of a binding government revenue constraint, we derive sufficient conditions for welfare improvement in the case where we implement (i) reforms in either production or consumption taxes, (ii) reforms in both consumption and production taxes and (iii) uniform changes in consumption taxes.

Keywords: Indirect tax reforms, Production and consumption generated pollution, Welfare, Government tax revenues.

J.E.L Classification: H21: Efficiency, Optimal Taxation
H23: Production taxes and subsidies

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1. Introduction

During the past couple of decades there has been a general consensus regarding the reforms of national tax systems. International institutions, e.g., the WTO, the IMF and the World Bank, encourage governments to reform their indirect tax structure.

Many types of reforms have been suggested. For example, the countries are urged to reduce their reliance on discriminatory trade taxes as tariffs, and switching to taxes such as income taxes, consumption taxes and VATs for the purpose of raising government revenues. Another suggested reform is to simplify the tax structure by bringing in more uniformity in it. Motivated by such developments in the policy arena, the academic literature identifies sufficient conditions under which proposed indirect tax reforms, e.g., reduction in trade taxes and increase in consumption taxes, or moving taxes towards uniformity, improve welfare and does not reduce government tax revenue. This latter concern becomes even more important for revenue strained developing economies. Achieving these two goals, countries are able to attain a so-called “double-dividend”. That is, a tax system which improves welfare and does not reduce tax revenues.

By now, a sizeable literature has addressed the aforementioned issues. In particular, within the context of open economies, two popular types of trade and/or domestic tax reforms have been examined. First, a policy of revenue-neutral reforms in trade taxes and/or in commodity taxes has been examined. Within this strand of the literature, studies such as Michael et al. (1993) identify sufficient conditions under which welfare improves when (i) tariffs decrease and consumption taxes increase while maintaining government revenue constant, and (ii) the total tax burden rate on goods moves towards uniformity, through adjustments either in consumption taxes or in tariffs, with or without a binding government revenue constraint. Abe (1995) identifies welfare improving sufficient conditions of a coordinated tariff and commodity tax reform in a small open economy with endogenous provision of public

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1 According to the World Bank (2002), during the 1990s in low- and medium-income countries, the share of domestic indirect taxes (i.e., taxes on goods and services) in total current government revenue rose from 26 percent in 1990 to 36 percent in 1999. During the same period the share of trade taxes fell from 17 percent to 9 percent.
2 Earlier literature on trade and domestic tax/subsidy reform policies, without a binding government revenue constraint include, among others, Hatta (1977a, 1977b), Diewer et al. (1989).
3 Other studies within this strand include works such as, Anderson (1999), Lahiri and Nasimi (2005).
goods. A second strand in the literature analyses a reform of trade taxes accompanied by appropriate changes in domestic taxes so that consumer or producer prices do not change. For example, Hatzipanayotou et al. (1994) demonstrate that welfare improves and government tax revenue increases when a uniform reduction in trade taxes is accompanied by appropriate increases in consumption taxes so that consumer prices remain constant. Keen and Ligthat (2002) generalize the Hatzipanayotou et al. (1994) result by demonstrating that welfare improves and government tax revenue increases with any tariff reduction that increases the value of domestic production at world prices, and is accompanied by a consumption tax reform which leaves consumer prices constant. Lahiri and Nasim (2005) examine the potential of revenue-neutral reforms of tariffs and sales taxes on final goods and intermediate inputs in Pakistan. They conclude that there is scope in reducing tariffs on final goods, but not on intermediate inputs. Emran (2005) considers selected reform strategies in a model of a small open economy with export taxes and taxes on production and consumption. Emran and Stiglitz (2005) conclude that the popular consensus requiring LDCs to reduce trade taxes and increase consumption (VAT) taxes in order to raise government revenue can be ineffective due to the existence of a sizable informal sector in these economies. Boadway and Sato (2007) extend the Emran-Stiglitz (2005) model by considering an economy with a formal and an informal sector, both producing only tradable, though different, outputs, whose production uses importable and exportable intermediate inputs. They investigate conditions under which one tax regime, e.g., a full VAT regime, is favored over the other, i.e., a full trade tax regime, as a way on increasing welfare and government tax revenues.

In the process of economic growth, one issue that worries policy makers is the impact of this expanded economic activity on the quality of environment. To this end, although by now there is a sizeable theoretical literature examining the links between economic expansion and environmental quality, there is only a limited number of studies which address the welfare and revenue implications of tax policy reforms in the context of pollution-ridden economies. Specifically, abstracting from government

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4 All the above studies examine the welfare and revenue implications of indirect tax reforms in the context of a static general equilibrium model of a small open economy. Yet, such tax reforms may also entail dynamic policy aspects, such as the growth rate of output (income). For such considerations see, among others, Naito (2005) and (2006).

5 As noted by the authors, if profits were fully taxed, then the VAT regime would be preferred to that of trade taxes. Emran-Stiglitz (2005) cynicism about the reforms has also been criticised by Keen (2006).
revenue considerations of tax policy reforms, Copeland (1994) identifies sufficient conditions for welfare-improving trade and environmental policy reforms in the context of a polluted small open economy. Beghin and Dessus (1999) examine the implications of reforms in trade and environmental policies on welfare and the level of pollution emissions under a government tax revenue constraint. Turunen-Red and Woodland (2004) examine selected Pareto-improving multilateral reforms of trade and production taxes in the context of a many countries and goods general equilibrium competitive model. Finally, Hatzipanayotou et al. (2005) examine the welfare implications of a number of multilateral environmental policy (pollution taxes) reforms in a two-country model of production generated cross-border pollution and of simultaneous provision of private and public sectors pollution abatement. 

To the best of our knowledge only a limited number of studies has, thus far, related the issue of tax policy reforms to consumption generated pollution. Beghin et al. (1997), abstracting from tax revenue considerations, examine the welfare implications of environmental, production, consumption, and trade tax policy reforms. Kayalica and Kayalica (2005) in a reciprocal dumping model with consumption generated pollution demonstrate, among other things, that a revenue neutral reform of increasing consumption taxes and reducing tariffs is strictly Pareto improving.

This paper considers a small open economy where pollution is generated either by production or by consumption. The government raises revenue and control pollution by imposing consumption and/or production taxes. Thus, we consider a more general model than what has been analyzed in the literature, and focus on two different types of indirect taxes rather than indirect tax and trade taxes. We also consider a situation when government revenue constraint is not binding as well as a situation when it is binding. Under these different scenarios, we derive sufficient conditions for welfare improvement in the case of specific types of reforms, viz., (i) increasing production (consumption) taxes and decreasing consumption (production) taxes, and (ii) reforms in production and consumption taxes.

\[\text{Naito (2005) examines in a dynamic context of a pollution ridden small open economy the welfare and growth implications of revenue-neutral tariff reforms.}\]

\[\text{Another strand of the literature, not however relevant for the present paper, examine economic implications of consumption generated pollution, e.g., Copeland and Taylor (1995), Perrings and Ansuategi (2000).}\]
2. The General Model

We consider a small open, perfectly competitive economy which produces and consumes \( K +1 \) internationally traded goods. There are \( K \) types of pollutants associated with the production or consumption of these goods. Good (0) is the *numeraire* good whose production does not generate any pollution. The country is endowed with the inelastic supply of \( M \) primary factors, denoted by the vector \( \bar{v} \).

Pollution is modeled as a by-product of both production and consumption. The production or consumption of each commodity generates a different type of pollutant. Let \( z_j \) and \( r_j \) (\( j = 1, 2, \ldots, K \)), denote respectively the level of pollution generated from the production and consumption of a unit of the \( j^{th} \) good. The levels of pollution \( z_j \) and \( r_j \) are soon explicitly defined. Production or consumption generated pollution adversely affects households’ utility. Consumption and production taxes are levied by the government to discourage respectively pollution-generating consumption by the country’s households and pollution-generating production by the producers. All tax revenues are lump-sum distributed to domestic households.

The country is a price taker in world commodity markets.\(^8\) The international prices of all goods are assumed to equal unity, and are denoted by the price vector \( p^* = (1, 1, \ldots, 1) \), a \((1 \times K)\) vector of unit-scalars.\(^9\) Thus, for the \( j^{th} \) commodity \( p_j = 1 + \tau_j \) is the domestic consumer price, and \( q_j = 1 - t_j \) be the domestic producer price, where \( \tau_j \) and \( t_j \) denote respectively the specific consumption and production tax levied on the \( j^{th} \) commodity. No taxes of any type are levied on the *numeraire* good(0), i.e., \( q_0 = p_0^* \).

The economy’s production side is represented by the revenue function \( R(1, q, \bar{v}) \) which captures the economy’s maximum revenue from production of the internationally traded goods with vector of factors \([\bar{v}]\)and vectors of producer prices \([1, q]\). For the rest of the analysis, since the vector of factor endowments \( \bar{v} \) remains unchanged, the revenue function is denoted by \( R(q) \). The \( R(q) \) function is

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\(^8\) We follow a standard practice of the literature of indirect tax reforms, which, by and large, for analytical convenience confines the analysis of such tax reforms in the context of small open economies, i.e., terms of trade considerations, are unaccounted for.

\(^9\) A prime (‘) denotes a transposed vector or matrix.
assumed convex and homogeneous of degree one in producer prices. By the envelop theorem \( R_{q_j} = \partial R / \partial q_j \) is the supply function of the \( j^{th} \) good. Production generated pollution is \( z_j = \alpha_j R_{q_j}(q) \), implying that the production of each good generates a different type of pollutant, and where \( \alpha_j > 0 \), is a scalar and denotes the units of pollution generated by the production of a unit of the \( j^{th} \) good.

Turning to the demand side of this economy, it comprises of identical households who consume the \( K+1 \) commodities, and whose utility is adversely affected by production and consumption generated pollution. A representative household’s preferences are captured by the expenditure function \( E(l, p, z, r, u) \) denoting the minimum expenditure on private goods achieving a certain level of utility \( u \), at consumer price vector \( p \) and vector of production and consumption pollutants \( z \) an d \( r \). We define the level of pollution generated by the consumption of a unit of the \( j^{th} \) good as \( r_j = \beta_j E_{p_j}(1, p, z, r, u) \), where \( \beta_j > 0 \) is a scalar. This specification again implies that the consumption of each good generates a different type of pollutant. The \( E(l, p, z, r, u) \) function is increasing in \( u \), in level of pollution \( z \) or \( r \), and non-decreasing and concave in \( p \).\(^{10}\) The derivative \( E_{p_j} = \partial E / \partial p_j \) is the compensated demand for good \( j \), and \( E_{p p} \) is a \((K \times K)\) negative semi-definite matrix.\(^{11}\) The derivative \( E_u \) captures the inverse of the marginal utility of income, and the derivative \( E_{z_j} \) or \( E_r \) is respectively, the marginal damage caused by the pollutant \( z_j \) or \( r \), and thus it represents the household’s *marginal willingness to pay* for its reduction (e.g., see Copeland, 1994).

The government’s tax revenue, \((T)\), which is distributed to households in a lump-sum fashion, equals the sum of consumption and production tax revenues. That is,

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\(^{10}\) The \( E(.) \) function is increasing in \( z \) or in \( r \) since an increase in any type of pollutant is assumed to harm the households’ utility. Therefore, to attain a given level of utility, \( u \), private spending on consumption must rise.

\(^{11}\) The compensated demand and supply functions for the *numeraire* good are respectively, \( E_{p_0} \) and \( R_{q_0} \).
\[ T = r'E_p(p, z, r, u) + r'R_q(q) = \sum_{j=1}^{K} \tau_j E_{p_j}(p, z, r, u) + \sum_{j=1}^{K} t_j R_{q_j}(q), \quad (1) \]

where \( E_p \) and \( R_q \), respectively, are the vectors of compensated demand and the output supply functions. Recall that for the \textit{numeraire} good\( (0) \), \( \tau_0 = t_0 = 0 \). The country’s income-expenditure identity requires that private spending on goods must equal income from production plus income from government taxes. Thus, the country’s budget constraint is given as follows:

\[ E(p, z, r, u) = R(q) + r'E_p(p, z, r, u) + r'R_q(q). \quad (2) \]

Equations (1) and (2) are the main equations of the model. They are used to examine the welfare implications of reforms in production and consumption taxes under two scenarios. First, we consider the case where there is no government revenue constraint. Second, we consider the case of a binding government revenue constraint by including an additional condition that \( dT=0 \).

We conclude this section by deriving the effects of the taxes on welfare and revenue levels. Differentiating equation (2), we obtain:

\[ du = -(\beta E_r - \tau)' dE_p - (\alpha E_z - t)' dR_q \quad (3) \]

where

\[ dE_p = E_{pp} \tau + E_{pr} dr + E_{pz} dz + E_{pu} du, \quad \text{and} \]

\[ dR_q = -R_{qq} dt. \quad (5) \]

For the rest of the analysis, we assume, for simplicity, that private goods and clean environment are independent in consumption, i.e., \( E_{pr} = E_{pz} = 0. \)

Equation (3) can be rewritten so as to capture the welfare effect of changes in a single consumption tax, say that on the \( i^{th} \) good, and of changes in a single production tax, say on the \( n^{th} \) good. That is:

\[ \text{12} \text{ The assumption that the demand for private goods is independent of the environmental quality is often made in the literature (i.e., Bovenberg 1999, Beghin and Dessus 1999). For example, this would be the case if the utility function is quasi-linear, e.g., } u(c, z) = \bar{u}(c) + \bar{\lambda} z, \text{ where } \bar{\lambda} \text{ is a constant parameter. Clearly, in this case goods and clean environment are independent in consumption.} \]
\[ \Omega \, du = -\sum_{j=1}^{K} (\beta_j E_{r_j} - \tau_j) E_{p_j, p_j} d\tau_j + \sum_{j=1}^{K} (\alpha_j E_{z_j} - t_j) R_{q_{j, q_{j}}} dt_n, \]  

(6)

where \( \Omega = E_u + \sum_{j=1}^{K} (\beta_j E_{r_j} - \tau_j) E_{p_j, p_j} \), and is normally assumed to be positive.\(^{14}\) It represents the general equilibrium inverse of the marginal utility of income; inclusive of feedback via consumption taxes and consumption generated pollution. Equation (6) can be further elaborated on by using the properties of the expenditure and revenue functions that compensated demands and output supplies are homogeneous of degree zero in prices. Specifically, \( \sum_{j=0}^{K} p_j E_{p_j, p_j} = 0 \) and \( \sum_{j=0}^{K} q_j R_{q_{j, q_{j}}} = 0 \), respectively, yield

\[ E_{p_j, p_j} = -\left( \frac{p_0}{p_j} \right) E_{p_0, p_0} - \sum_{j=0}^{K} \left( \frac{p_j}{p_i} \right) E_{p_j, p_i} \quad \text{and} \quad R_{q_{j, q_{j}}} = -\left( \frac{q_0}{q_j} \right) R_{q_0, q_0} - \sum_{j=0}^{K} \left( \frac{q_j}{q_i} \right) R_{q_j, q_i}. \]

Note that \( p_k = 1 + \tau_k = 1 - t_k \), \( k = j, i, n \), and by the reciprocity conditions \( E_{p_j, p_j} = E_{p_j, p_i} \) and \( R_{q_j, q_j} = R_{q_j, q_i} \). Using the above properties and after some manipulations, we obtain:

\[ \Omega \, du = \left[ \sigma_i E_{p_i, p_i} + \sum_{j=0}^{K} (\sigma_i - \sigma_j) p_j E_{p_j, p_j} \right] d\tau_i + \left[ -s_n R_{q_{j, q_j}} + \sum_{j=0}^{K} (s_j - s_n) q_j R_{q_j, q_j} \right] dt_n. \]  

(7)

We shall call the ratio \( \sigma_k = (\beta_k E_{r_k} - \tau_k) / p_k > 0(<0) \) the rate of under-taxation of consumption-pollution when \( \sigma_k > 0 \), and the rate of over-taxation of consumption-pollution when \( \sigma_k < 0 \). That is, if the marginal willingness to pay for the pollution reduction for the \( k^{th} \) good is greater than its pollution tax, then this good is under-

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\(^{13}\) In this case, equation (3) is \( du = -\sum_{j=1}^{K} (\beta_j E_{r_j} - \tau_j) dE_{p_j} - \sum_{j=1}^{K} (\alpha_j E_{z_j} - t_j) dR_{q_j} \). Simple algebra, using the relevant equations (4) and (5), and assuming that \( E_{p_j, p_j} = E_{p_j, p_i} = 0 \), result in equation (6).

\(^{14}\) Subscripts on the functions, i.e., \( E_{p_j, p_j}, E_{p_j, p_i}, E_{p_j, p_k} \) and \( R_{q_{j, q_j}}, R_{q_{j, q_i}} \), denote partial derivatives. For example, \( E_{p_j, p_i} = \partial E_{p_j} / \partial p_i, \quad R_{q_{j, q_j}} = \partial R_{q_j} / \partial q_n \). It is to be noted that \( E_{p_j, p_i} > 0(<0) \) if the \( j^{th} \) and \( i^{th} \) goods are substitutes (complements) in consumption, \( E_{p_j, p_k}, \forall j \in K \), is positive assuming that all goods are normal in consumption, and \( R_{q_{j, q_j}} < 0(>0) \) if the \( j^{th} \) and \( n^{th} \) goods are substitutes (complements) in production.
taxed and vise versa.\textsuperscript{15} Similarly, the ratio \( s_k = (\alpha_k E_{z_k} - t_k) / q_k, k = j, n \), is positive (negative) depending on whether the \( k^{th} \) production generated pollutant is under (over-) taxed. For the purposes of our analysis, we call \( s_k > 0 \) the rate of under-taxation of production-pollution, and \( s_k < 0 \) the rate of over-taxation of production-pollution. Since it is assumed that the numeraire good, is non-polluting and untaxed, \( \sigma_0 = (\beta_0 E_{n_0} - \tau_0) / p_0 = 0 \) and \( s_0 = (\alpha_0 E_{z_0} - \tau_0) / q_0 = 0 \).

When government revenue constraint is binding \((dT = 0)\), differentiating equation (2), using equations (4), (5), and the homogeneity properties of the expenditure and revenue functions, we obtain:

\[
\delta du + \left( 1 + \tau_i (1 - \eta_{i0}) \right) E_{p_i} + \sum_{j \neq i, 0} \left( \tau_j - \tau_i \right) E_{p_j p_i} \right) \frac{d\tau_i}{1 + \tau_i} + \sqrt{\sum_{j \neq i, 0} \left( t_n - t_j \right) R_{q_{ij}} \right) \frac{dt_n}{1 - t_n} = 0 , \tag{8}
\]

where, \( \delta = \sum_{j=1} \tau_j E_{p_j} \) and it is positive assuming that goods are normal in consumption; \( \eta_{i0} = (p_0 / E_{p_i}) E_{p_j p_n} \) is the compensated demand elasticity of the \( i^{th} \) good with respect to the consumer price of the numeraire, \( \varepsilon_{n0} = (q_0 / R_{q_i}) R_{q_{in}} \) is the elasticity of supply of the \( j^{th} \) good with respect to the producer price of the numeraire.\textsuperscript{16}

Equations (3) and (7) are relevant for examining the welfare implications of the indirect tax reforms assuming a non-binding government revenue constraint. The system of equations (3), (7) and (8) are used to examine the welfare implications of

\textsuperscript{15} Note that \( p_k^{-1} E_{\tau_k} \beta_k = p_k^{-1} (\partial E / \partial r_k) (\partial r_k / \partial E_{n_k}) \) is the amount by which consumers need to be compensated in order to keep utility constant due pollution generated by a Euro’s (dollar’s) worth increase in consumption of the \( k^{th} \) good. \( \tau_k / p_k \) is the ad-valorem equivalent of the specific consumption tax \( \tau_k \) on the \( k^{th} \) good.

\textsuperscript{16} The term \( (1 + \tau_i (1 - \eta_{i0})) E_{p_i} \) emerges following straightforward algebra of \( (p_i E_{p_i} - \tau_i E_{p_i p_n}) \). Likewise manipulating the term \( (q_n R_{q_n} + t_n R_{q_{in}}) \) results in \( (1 - t_n (1 - \varepsilon_{n0})) R_{q_n} = [1 - t_n (1 - \varepsilon_{n0})] R_{q_n} \).
indirect tax reforms under a binding government revenue constraint and in the presence of both consumption-generated and production-generated pollution.

3. Absence of Government Revenue Constraints

In this section, we assume away the existence of any government revenue constraints and examine the welfare implications of reforms in consumption taxes and in production taxes. We consider these one at a time, but in the presence of both types of pollution and both types of taxes.

3.1 Reforms in consumption taxes

In this subsection, we examine the welfare implications of increasing (decreasing) the consumption tax on the good which exhibits the highest rate of consumption-pollution under-taxation (over-taxation), i.e., we shall increase (decrease) the consumption tax rate for the good if \( \sigma_i - \sigma_j < 0(>0) \), to the point where \( \sigma_i \) falls (rises) towards the level of the second highest (lowest) \( \sigma \). In this exercise we do not consider changes in production taxes whose non-zero levels are held constant. With this in mind, whether there exist production generated pollution and/or production taxes does not affect the results to follow. Since production taxes do not change, equation (7) reduces to:

\[
\Omega \ du = -\sum_{j=1}^{K} (\beta_j E_{j} - \tau_j) E_{j,p;\beta} d\tau_j = \left[ \sigma_i E_{j,p;\beta} + \sum_{j=1}^{K} (\sigma_i - \sigma_j) p_j E_{j,p;\beta} \right] d\tau_i. \tag{9}
\]

The following proposition which is derived directly from equation (9), states sufficient conditions for welfare improving consumption tax reforms required for moving the rates of under-taxation or over-taxation of consumption-pollution towards uniformity.

Proposition 1: Assume the existence of consumption and production generated pollution, and that some goods are under-taxed while some other are over-taxed.

- Suppose that the \( i^{th} \) good exhibits the highest rate of under-taxation of consumption-pollution, i.e., \( \sigma_i > 0 \) and \( (\sigma_i - \sigma_j) > 0, \forall j \in K \). Then, increasing the consumption tax on this good, so that its rate of under-taxation of consumption-pollution does not fall below the level of the second highest
under-taxation rate, improves social welfare if the $i^{th}$ good is a substitute in consumption with all other goods.

- Suppose that the $i^{th}$ good exhibits the highest rate of over-taxation of consumption-pollution, i.e., $\sigma_i < 0$ and $(\sigma_i - \sigma_j) < 0, \forall j \in K$. Then decreasing the consumption tax on this good, so that its rate of over-taxation of consumption-pollution does not fall below the level of the second highest rate of over-taxation, improves welfare if, in consumption, the $i^{th}$ good is a substitute to all other goods.

Intuitively, the above results can be interpreted as follows. Take the case whereby the $i^{th}$ good exhibits the highest rate of consumption-pollution under-taxation, thus it is the good associated with the most distorted consumption-pollution. Then, increasing the consumption tax on this good so that its rate of under-taxation of consumption-pollution does not fall below the level of the second highest rate, aims at bringing the consumption generated pollution distortions towards uniformity. This result depends on the relationship in consumption between the good with the highest rate of under-taxation of consumption-pollution, and of all other goods, including the numeraire good. Thus, assuming substitutability in consumption between the good with the highest rate of under-taxation of consumption-pollution and all other goods, an increase in the consumption tax on this good reduces its consumption and pollution distortion and raises the consumption and pollution distortion generated by all other goods. An analogous argument holds when the $i^{th}$ good exhibits the highest rate of over-taxation of consumption-pollution, and the consumption tax on this good is reduced in such a way that, its rate of over-taxation of consumption-pollution does not fall below the level the second highest rate

### 3.2 Reforms in production taxes

Next we examine the welfare implications of increasing (decreasing) the production tax on the good which exhibits the highest rate of production-pollution under-taxation (over-taxation), i.e., we reduce (increase) the $n^{th}$ production tax rate when $(s_j - s_n) < 0(> 0), \forall j$. In this exercise we do not consider changes in consumption taxes whose non-zero levels are held constant. Whether there exist
consumption generated pollution and/or consumption taxes does not affect the results to follow. Since consumption taxes do not change here, equation (7) reduces to:

\[
\Omega \, du = \left[-s_n R_{\theta,\rho,\epsilon} + \sum_{j \in A, \beta} (s_j - s_n) q_j R_{q,\rho,\epsilon} \right] dt_n. \tag{10}
\]

The following proposition which follows directly from equation (10), summarizes the results of this reform program.

**Proposition 2:** Assume the existence of consumption-generated and production-generated pollution, and that some goods are under-taxed while some others are over-taxed.

- **Suppose the** \(n^{th}\) **good exhibits the highest rate of under-taxation of production-pollution, i.e.,** \(s_n > 0\) **and** \((s_j - s_n) < 0, \forall j \in K\). **Then increasing the production tax on this good in a way that its rate of production-pollution under-taxation does not fall below of the second highest rate, improves welfare if the** \(n^{th}\) **good is a substitute in production to all other commodities.**

- **Suppose the** \(n^{th}\) **good exhibits the highest rate of over-taxation of production-pollution, i.e.,** \(s_n < 0\) **and** \((s_j - s_n) > 0, \forall j \in K\). **Then decreasing the production tax on this good in a way that its rate of over-taxation of production-pollution does not fall below of the second highest rate, improves welfare if the** \(n^{th}\) **good is a substitute in production to all other goods.**

In the presence of pollution, Proposition 2 identifies some key conditions for welfare improving reforms in production taxes which move towards uniformity the rates under-(over-) taxation of production-pollution. The intuition of these results can be as follows. When, for example, the \(n^{th}\) taxed good exhibits the highest rate of under-taxation of pollution, it generates the most production related pollution distortion. Then, increasing the production tax on this good such that its rate of under-taxation of pollution does not fall below of the second highest rate, it aims at bringing production generated environmental distortions towards uniformity. This result

\[^{17}\text{The size of } \Omega \text{ is different if consumption taxes are zero compared to the case where are not. The results of proposition 2, however, are the same in both cases, i.e., zero or positive consumption taxes.}\]
depends on the relationship in production between the good with the highest rate of production-pollution under-taxation, and all other goods, including the *numeraire* commodity. Thus, assuming substitutability in production between the good with the highest rate of pollution under-taxation and all other goods, an increase in the production tax on this good, reduces its production and pollution distortion and raises the production and pollution distortion of all other goods. An analogous argument holds when the $n^{th}$ good exhibits the highest rate of over-taxation of pollution, and a decrease in the production tax on this good, assuming substitutability in production between this good and all other commodities, moves its rate of over-taxation of pollution closer to the second highest rate.18

3.3 Uniform changes in consumption taxes

In this subsection we investigate the possibility of a welfare improving uniform increase/decrease in consumption taxes. In particular, we consider a change in consumption taxes by the same proportion $(0 < \lambda < 1)$ of the rate of consumption-pollution under taxation. That is, let $d\tau_i = \lambda(\beta E_\gamma - \tau_i)$, where $d\tau_i > (<)0$ according to whether $(\beta E_\gamma - \tau_i) > (<)0$. That is, the tax on consumption of the $i^{th}$ polluting commodity is raised (lowered) according to whether pollution emissions are socially under (over)-taxed. For this reform, equation (3) can be written as:

$$\Omega \ du = -\lambda(\beta E_r - \tau)^\epsilon E_{pE}(\beta E_r - \tau) > 0.$$  (11)

The following proposition states the above result formally.

**Proposition 3:** A uniform increase (decrease) in consumption taxes proportional to the difference between the marginal willingness to pay for pollution generated by consumption and the actual tax on this good, improves welfare.

---

18 When the pollution from the production of different goods is homogenous and pollution intensities are also the same, then the rate of under-taxation of pollution is the highest (i.e., $\sigma_i$ or $s_i$ is the highest) if and only if the tax rate is the lowest (i.e., $\tau_i$ or $t_i$ is the lowest). Similarly, the rate of over-taxation of pollution on a good is the highest if the tax rate of this good is the highest.
From the discussion of equations (9)-(11) it is important to note that, in the present context of pollution, what is required is the reform of consumption and/or production taxes so that the rates of under (over-) taxation of consumption pollution, or of production-pollution move towards uniformity. Thus, contrary to indirect tax reforms considered in the literature, e.g., reforms of consumption taxes and tariffs (Michael et al., 1993, Hatta 1977), the present reform exercise may not have any bearing on whether the actual production or consumption tax rates move or diverge from uniformity. 19

4. Reforms under a binding revenue constraint

In this section we consider reforms in consumption and production taxes under the additional restriction that government revenue cannot change because of the reforms. Thus, contrary to the previous section, we can no longer consider a change in a single consumption or production tax. In other words, we need to consider changes in at least two of these taxes in order to keep government revenue unchanged. Accordingly, we shall consider three reforms in the following three subsections: (i) changing one production tax and one consumption tax, (ii) changing two production taxes, and (iii) changing two consumption taxes. These three cases are now taken up in turn.

4.1 Reforms in consumption and production taxes

Equations (7) and (8) are now used to examine the welfare implications of the aforementioned reform programs, as well as the required adjustments in tax rates in order to maintain government revenue constant. To facilitate the analysis, we rewrite equations (7) and (8) as follows:

\[ \Omega \ du = p_i^{-1} F_i \ d\tau_i + q_n^{-1} B_n \ dt_n, \]
\[ \delta du + p_i^{-1} G_i \ d\tau_i + q_n^{-1} D_n \ dt_n = 0, \]

\[ \Omega \ du = p_i^{-1} F_i \ d\tau_i + q_n^{-1} B_n \ dt_n, \]
\[ \delta du + p_i^{-1} G_i \ d\tau_i + q_n^{-1} D_n \ dt_n = 0, \]

19 It can be easily shown that, in the present context, previous results of the standard literature of tariffs and consumption tax reforms go through only in the unlikely case of reforming consumption taxes but in the presence of production generated pollution. In such an unlikely case, if, for example, the \( i^{th} \) good is burdened with the highest (lowest) consumption tax rate, then, reducing (increasing) this tax rate to the level of the next highest (lowest) consumption tax rate, unambiguously improves the country’s welfare if the \( i^{th} \) good is a substitute to all other goods in consumption (see, e.g., Michael et al., 1993, Proposition 1, p. 421).
where,
\[ F_i = p_i \left[ \sigma_i E_{p_i} + \sum_{j=1}^{n} p_j (\sigma_j - \sigma_i) E_{p_j} \right], \quad G_i = \left[ (1 + \tau_i (1 - \eta_i)) E_{p_i} + \sum_{j=1}^{n} (\tau_j - \tau_i) E_{p_j} \right], \]
\[ B_n = q_n \left[-s_n R_{q_n} + \sum_{j=n+1}^{n} (s_j - s_n) q_j R_{q_j}\right], \quad D_n = \left[ \left(1 - t_n (1 - \varepsilon_n) \right) R_{q_n} + \sum_{j=n+1}^{n} (t_j - t_n) R_{q_j}\right]. \]

We rewrite equations (12) and (13) in the following matrix format:

\[
\begin{bmatrix} \Omega & -p_i^{-1} F_i \\ \delta & p_i^{-1} G_i \end{bmatrix} \begin{bmatrix} du \\ d\tau_i \end{bmatrix} = \begin{bmatrix} q_n^{-1} B_n \\ -q_n^{-1} D_n \end{bmatrix} dt_n.
\]

(14)

Solving the above equation, we obtain:

\[
\Delta \left( \frac{du}{dt_n} \right) = \left( p_i q_n \right)^{-1} \left( B_n G_i - F_i D_n \right).
\]

(15)

\[
\Delta \left( \frac{d\tau_i}{dt_n} \right) = -q_n^{-1} (\Omega D_n + \delta B_n),
\]

(16)

where \( \Delta = p_i^{-1} (\Omega G_i + \delta F_i) \) is the determinant of the left-hand-side coefficients matrix in (14) and it is positive assuming that the consumption tax rate \( \tau_i \) is revenue increasing.\(^{20}\)

Equation (16) indicates that increasing the production tax rate \( t_n \) reduces the consumption tax \( \tau_i \), i.e., \( (d\tau_i / dt_n) < 0 \), assuming that \( t_n \) is a revenue increasing production tax.\(^{21}\) Thus, in order to keep government revenue unchanged, the two taxes need to move in the opposite direction.

\(^{20}\) In equations (14) and (15) treating \( du \) and \( dT \) as endogenous and \( d\tau_i \) and \( dt_n \) as exogenous, it can be shown that \( (dT / d\tau_i) = \Omega^{-1} p_i^{-1} (\Omega G_i + \delta F_i) \). Thus, \( (dT / d\tau_i) > 0 \) requires that \( (\Omega G_i + \delta F_i) \) is positive.

\(^{21}\) Similarly, it can be shown that \( (dT / dt_n) = \left( q_n \Omega \right)^{-1} (\Omega D_n + \delta B_n) \). Then, \( (dT / dt_n) \) is positive if \( (\Omega D_n + \delta B_n) \) is positive.
The following proposition summarizes the conditions ensuring a welfare improvement due to an increase in the production tax \( t_n \), adjusting appropriately the consumption tax \( \tau_i \), so that government revenue is held constant.

**Proposition 4:** Assume the existence of production and consumption generated pollution, some goods are under-taxed while some others are over-taxed, and that

(i) the \( n^{\text{th}} \) good exhibits the highest rate of under-taxation of production-pollution, i.e., \( s_n > 0 \) and \( (s_j - s_n) < 0, \forall j \in K \), it has the lowest production tax, i.e., \( t_n < t_j \forall j \in K \), and it is a substitute to all other goods in production,

(ii) in absolute value the cross-price elasticity of supply of the \( n^{\text{th}} \) good with respect to the price of the numeraire is less than \( (1 - t_n)/t_n \), (i.e., \( -\varepsilon_{n0} < (1 - t_n)/t_n \)), \(^{22}\)

(iii) the \( i^{\text{th}} \) commodity exhibits the highest rate of over-taxation of consumption-pollution, i.e., \( \sigma_i < 0 \) and \( (\sigma_i - \sigma_j) < 0, \forall j \in K \), it has the highest consumption tax, i.e., \( \tau_i > \tau_j \forall j \in K \), and it is a substitute to all other goods in consumption,

(iv) \( \tau_i \) is a revenue increasing consumption tax rate.

Then, a small increase in the production tax on the \( n^{\text{th}} \) good in such a way that it does not exceed the second lowest and the rate of under-taxation of production-pollution does not fall below of the second highest rate, while reducing the consumption tax on the \( i^{\text{th}} \) good to keep government revenue constant, increases social welfare.

For the increase in the production tax \( t_n \) to raise welfare the right-hand-side term of equation (15) must be positive. Condition (i) of Proposition 4 ensures that \( B_n \) is positive. Conditions (i) and (ii) ensure that \( D_n \) is positive, and condition (iii) ensures that \( F_i \) is negative. Finally, since, by condition (iv), the determinant \( \Delta \) is

\(^{22}\) This condition is almost certain that holds since the \( n^{\text{th}} \) good is the good with the lowest production tax.
positive, $\Omega$ is positive by the required stability conditions, and $F_i$ is negative, then $G_i > 0$. Thus, $(du/dt_n)$ is positive.

Following the above analysis, consider the case where the $n^{th}$ good exhibits the highest rate of over-taxation of production-pollution, and the $i^{th}$ good exhibits the highest rate of under-taxation of consumption-pollution. Then, conditions similar to (i)-(iii) of Proposition 4 and that $t_n$ is a revenue increasing production tax, suffice to ensure an improvement in welfare when reducing the production tax on the $n^{th}$ good and increasing the consumption tax on the $i^{th}$ good so that government revenue is held constant.

Finally, by the same procedure, one can easily examine the welfare implications of consumption tax reforms (i.e., changes in $\tau$) while appropriately adjusting the production tax $t_n$ so as to maintain constant government tax revenue. For example, from equations (14) we can obtain:

$$\left(\frac{du}{d\tau_i}\right) = -\left(\Delta_i, \eta_i q_n\right)^{-1}(B_i G_i - F_i D_n) < 0 \text{ and } \left(\frac{dt_n}{d\tau_i}\right) = -(\Delta_i q_n)^{-1}(\Omega G_i + \delta F_i) < 0, \quad (17)$$

where $\Delta_i = (\Omega D_n + \delta B_n)$, as shown in footnote (19), is positive assuming that $t_n$ is a revenue increasing production tax. Equations (17) indicate that under the assumptions of the model and conditions similar to ones previously described, a reduction of the consumption tax $\tau_i$, so as the highest rate of under-taxation of consumption-pollution of this good does not fall below of the second highest rate, and an appropriate increase in the lowest production tax rate $t_n$ improves the country’s welfare and maintain constant the government revenue.

Next, assuming the existence of production and consumption generated pollution we consider two special cases of the above general results. First, under the constraint of constant government revenue, we examine the welfare implications of moving the rates of under (over-) taxation of production-pollution towards uniformity via reforms in production taxes. Second, we examine, under the constraint of constant government revenue, the welfare implications of moving the rates of under (over-)
taxation of consumption-pollution towards uniformity via reforms in consumption taxes.

4.2 Reforms in production taxes

In this section, we consider changes in two production taxes, *viz.*, for the \( n^{th} \) and the \( i^{th} \) good. In this case, we obtain:

\[
\Delta_2 \left( \frac{dt_i}{dt_n} \right) = -q_n^{-1} \left[ (1-\delta)D_n + \delta B_n \right], \quad \text{and} \quad (18)
\]

\[
\Delta_2 \left( \frac{du}{dt_n} \right) = (q_i q_n)^{-1} \left[ B_n D_i - B_i D_n \right], \quad (19)
\]

where \( \Delta_2 = q_i^{-1} \left[ (1-\delta)D_i + \delta B_i \right] \) and it is positive assuming that \( t_i \) is a revenue increasing production tax. Appendix (A.1) provides the relevant algebra in deriving the above equations.

The right-hand-side term of equation (18), i.e., \( q_n^{-1} \left[ (1-\delta)D_n + \delta B_n \right] \), is positive assuming that \( t_n \) is revenue increasing tax.\(^{23}\) Thus, equation (18) indicates that for tax revenue to remain constant, the increase in \( t_n \), must be accompanied by a reduction in the production tax \( t_i \), assuming that both rates are revenue increasing taxes. Thus, \( (dt_i / dt_n) < 0 \). That is, once again changes in the two tax rates have to be in the opposite direction in order for the government revenue to remain unchanged. In equation (19), the expressions \( B_i \) and \( D_i \) for the \( i^{th} \) good are similar to those for the \( n^{th} \) good. The following proposition states the sufficient conditions for a welfare improving increase in \( t_n \), when \( t_i \) is reduced so that tax revenue remains constant.

**Proposition 5:** Assume the existence of production generated pollution, that some goods are under-taxed while some are over-taxed, and that

(i) the \( n^{th} \) good is a substitute to all other goods in production, it exhibits the highest rate of under-taxation of production-pollution, i.e., \( s_n > 0 \) and

\(^{23}\) With changes only in production taxes alone, it can be shown that \( (dT / dt_n) = (q_n (1-\delta))^{-1} \left[ (1-\delta)D_n + \delta B_n \right] \). Therefore, for \( (dT / dt_n) \) to be positive, it is required that \( \left[ (1-\delta)D_n + \delta B_n \right] \) is positive.
\[(s_j - s_n) < 0, \forall j \in K, \text{ and it has the lowest production tax, i.e., } t_n < t_j, \forall j \in K.\]

(ii) \[-\varepsilon_{n0} < (1 - t_n)/t_n.\]

(iii) the \(i^{th}\) good exhibits the highest rate of over-taxation of production-pollution, i.e., \(s_n < 0\) and \((s_j - s_i) > 0, \forall j \in K, \text{ and it is a substitute in production to all other goods in production}\)

(iv) \(t_i\) is a revenue increasing production tax rate

Then, a small increase in the production tax on the \(n^{th}\) good in such a way that the rate of under-taxation of production-pollution does not fall below of the second highest, and reducing the production tax on the \(i^{th}\) good to keep government revenue constant, increases social welfare.

For the increase in the production tax \(t_n\) to raise welfare the right-hand-side term of equation (19) must be positive. Condition (i) of Proposition 5 ensures that \(B_n\) is positive, conditions (i) and (ii) ensure that \(D_n\) is positive, and condition (iii) ensures that \(B_i\) is negative. Since the determinant \(\Delta_2\) is positive by condition (iv), \(\delta\) is positive by assumption, and \(B_i < 0\), then \(D_i\) is positive. Therefore, \((du/dt_n) > 0\).

4.3 Reforms in consumption taxes

In this subsection, we consider changes in two consumption taxes and the relevant two equations can be obtained as follows:

\[
\Delta \left( \frac{du}{d\tau_n} \right) = (p_i p_n)^{-1} (F_n G_i - F_i G_n). \\
(20)
\]

\[
\Delta \left( \frac{d\tau_i}{d\tau_n} \right) = -p_n^{-1} (\Omega G_n + \delta F_n). \\
(21)
\]

Appendix (A.2) provides the relevant algebra in deriving the above equations.

Equation (21) indicates that an increase in the consumption tax rate \(\tau_n\) reduces the consumption tax \(\tau_i\), i.e., \((d\tau_i / d\tau_n) < 0\), assuming that the \(n^{th}\) consumption tax is
revenue increasing.\footnote{Following footnote (21), it can be shown that $\left(\frac{dT}{dT_n}\right)$ is positive if $\left(\Omega G_n + \delta F_n\right)$ is positive.} That is, the two tax rates need to move in the opposite direction in order to keep government revenue unchanged.

The following proposition summarizes the sufficient conditions, according to equation (20), ensuring a welfare improvement due to an increase in the consumption tax $\tau_n$, adjusting appropriately the consumption tax $\tau_j$, so that government revenue is held constant.

**Proposition 6:** Assume the existence of consumption generated pollution, that some goods are under-taxed while some are over-taxed, and let:

- (i) the $n^{th}$ good exhibit the highest rate of under-taxation of consumption-pollution, i.e., $\sigma_n > 0$ and $(\sigma_n - \sigma_j) > 0, \forall j \in K$, has the lowest consumption tax, i.e., $\tau_n < \tau_j, \forall j \in K$, and be a substitute to all other goods in consumption,
- (ii) the elasticity of compensated demand for the $n^{th}$ good with respect to changes in the price of the numeraire be less than $(1 + \tau_n)/\tau_n$.
- (iii) the $i^{th}$ good exhibit the highest rate of over-taxation of consumption-pollution, i.e., $\sigma_i < 0$ and $(\sigma_i - \sigma_j) < 0, \forall j \in K$, and be a substitute to all other goods in consumption, and
- (iv) $\tau_i$ be a revenue increasing consumption tax rate.

Then a small increase in the consumption tax on the $n^{th}$ good in such a way that the rate of under-taxation of consumption-pollution does not fall below of the second highest rate while decreasing the consumption tax rate on the $i^{th}$ good so as to keep government revenue constant, improves welfare.

For the increase in the consumption tax $\tau_n$ to raise welfare the right-hand-side term of equation (20) must be positive. Condition (i) of Proposition 6 ensures that $F_n$ is positive. Conditions (i) and (ii) ensure that $G_n$ is positive, while condition (iii) ensures that $F_i$ is negative. Since, $\delta$ and $\Omega$ are positive, by the required stability
conditions, \( \Delta = p_i^{-1}(\Omega G_i + \delta F_i) \) is positive, by condition \((iv)\), and \( F_i \) is negative, by condition \((iii)\), then \( G_i \) must be positive. Therefore, \( (du/d\tau_i) > 0 \).

6. Concluding Remarks

Recent developments in the theory and practice of economic policy making acknowledge the adverse consequences of expanded economic activity on the quality of environment. Such environmental degradation must then be accounted for when evaluating the welfare and other economic effects of various economic policies. With this in mind, we note that the literature on tax reforms within an integrated system of indirect taxes (e.g., VATs, or other domestic or trade taxes) offers, thus far, a very limited insight on the welfare and government revenue implications of such tax reforms in the presence of pollution ridden economies. Thus, in this paper we revisit the question of reforming the structure of indirect taxes in the presence of production and consumption-generated pollution, and we identify sufficient conditions under which such tax reforms improve welfare with and without a binding government revenue constraint.

The sufficient conditions under which the various tax reforms improve welfare with or without constant government revenue are stated in the relevant Propositions of the paper. Here, instead of restating these conditions, we note some analytical features related to our results. First, the presence of production generated pollution does not alter the known results of consumption tax reforms alone. Second, regardless of a binding revenue constraint, the proposed welfare improving reforms of production taxes alone, or of consumption and production taxes combined, are those bringing towards uniformity the rates of under (over-) taxation of pollution. The same feature holds for the case of consumption generated pollution and of reforming consumption taxes so as to bring the rates of under (over-) taxation of pollution towards uniformity. For example, consider the case of reforming production taxes alone. When there is no binding revenue constraint, a welfare improving reform entails increasing (decreasing) the production tax on the commodity exhibiting the highest rate of under (over-) taxation of pollution in a way such that this rate does not falls below the second highest rate of under (over-) taxation of pollution. When there is a binding revenue constraint, such a reform is accompanied by appropriate changes in the production tax on another commodity so that government revenue is kept constant.
Third, regardless of the source of pollution, two of the critical conditions supporting the results are: (i) the relationship in consumption and/or production between the good whose tax is changed to all other commodities, and (ii) under a binding revenue constraint, all reformed taxes are revenue increasing. Lastly, in the case of consumption generated pollution, a uniform increase (decrease) in consumption taxes proportional to the deviation between the marginal willingness to pay for pollution generated by consumption of the a good and the tax levied on it, improves welfare. An equivalent result can be easily shown for the case of production generated pollution, and of a uniform increase (decrease) in production taxes proportional to the deviation between the marginal willingness to pay for pollution generated by production of the a good and the tax levied on it. This result is closely related to a well known result, viz. Copeland (1994), of tax reforms in polluted small open economies. That is, in the presence of tariffs and abstracting from revenue considerations, a uniform increase (decrease) in production taxes proportional to the pollution distortion vector does not reduce welfare. 

25 Here, the pollution distortion vector consists of the deviations between the marginal willingness to pay for production generated pollution of each commodity and the tax levied on it. In Copeland (1994), due to the presence of tariffs, the pollution distortion of a good in addition to the above deviation it includes a third component accounting for the effect of the tariff distortion on the cost of pollution to consumers.
APPENDIX

A.1 Reforms in production taxes under pollution and a binding revenue constraint

With changes only in production taxes \( t_a \) and \( t_i \), equations (7) and (8) respectively, become:

\[
\begin{align*}
\Omega \ du &= \left[ -s_i R_{q_i q_i} + \sum_{j \neq i, b} (s_j - s_i) q_j R_{q_j q_i} \right] dt_i + \left[ -s_n R_{q_n q_n} + \sum_{j \neq n, b} (s_j - s_n) q_j R_{q_j q_n} \right] dt_n, \\
\delta du + \left[ (1-t_i(1-\varepsilon_{v,0})) R_{q_i} + \sum_{j \neq i, 0} (t_j - t_i) R_{q_j q_i} \right] dt_i + \\
&\quad \left[ (1-t_n(1-\varepsilon_{v,0})) R_{q_n} + \sum_{j \neq n, 0} (t_n - t_j) R_{q_j q_n} \right] dt_n = 0. \quad (A.1)
\end{align*}
\]

Equations (A.1) can be written in the following matrix system:

\[
\begin{bmatrix}
\Omega & -q_i^{-1} B_i \\
\delta & q_i^{-1} D_i
\end{bmatrix}
\begin{bmatrix}
du \\
dt_i
\end{bmatrix}
= \begin{bmatrix}
q_i^{-1} B_n \\
-q_n^{-1} D_n
\end{bmatrix}
dt_n, \quad (A.2)
\]

where the definitions for \( B_i, B_n, D_i \), and \( D_n \) follow those given in equations (12) and (13). Equations (A.2) are then used to derive equations (18) and (19) in the text.

A.2 Reforms in consumption taxes under pollution and a binding revenue constraint

With changes only in consumption taxes \( \tau_a \) and \( \tau_i \), equations (7) and (8) respectively, become:

\[
\begin{align*}
\Omega \ du &= \left[ \sigma_a E_{p_i p_i} + \sum_{j \neq i, 0} (\sigma_j - \sigma_i) p_j E_{p_j p_i} \right] \! dt_i + \left[ \sigma_n E_{p_n p_n} + \sum_{j \neq n, 0} (\sigma_n - \sigma_j) p_j E_{p_j p_n} \right] \! dt_n, \\
\delta du + \left[ (1+\tau_i(1-\eta_{v,0})) E_{p_i} + \sum_{j \neq i, 0} (\tau_j - \tau_i) E_{p_j p_i} \right] \! \frac{dt_i}{1+\tau_i} + \\
&\quad \left[ (1+\tau_n(1-\eta_{v,0})) E_{p_n} + \sum_{j \neq n, 0} (\tau_j - \tau_n) E_{p_j p_n} \right] \! \frac{dt_n}{1+\tau_n} = 0. \quad (A.3)
\end{align*}
\]

Equations (A.3) can be written in the following matrix system:
\[
\begin{bmatrix}
\Omega & -p_i^{-1} F_i \\
\delta & p_i^{-1} G_i
\end{bmatrix}
\begin{bmatrix}
du \\
d\tau_i
\end{bmatrix} =
\begin{bmatrix}
p_n^{-1} F_n \\
-p_n^{-1} G_n
\end{bmatrix} d\tau_n,
\]  
(A.4)

where the definitions for $F_i, F_n, G_i$ and $G_n$ follow those given in equations (12) and (13). Equations (A.4) are then used to derive equations (20) and (21) in the text.
References
Hatta, T., 1977a, A recommendation for a better tariff structure, Econometrica 45, 1859-1869.
Keen, M. 2006, VAT, tariffs and withholding: Border taxes and the informality in developing countries, mimeo, International Monetary Fund, Washington, D.C.