Infrastructure Provision and Macroeconomic Performance

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Abstract

Behavioral differences between economies where infrastructure is privately provided and where the government is the sole provider are examined in the context of a growing economy. The choice between private and public provision generates differences in the private sector’s ability to internalize capital utilization decisions and market prices along the equilibrium path. This in turn has a crucial impact on the effects of fiscal policy on resource allocation and welfare in each regime. If the government wants to stimulate infrastructure investment, a subsidy to private providers yields significantly higher welfare gains than an equivalent increase in direct government investment, even with lump-sum tax financing. On the other hand, an income tax is more distortionary under private than under government provision. In designing optimal fiscal policy, while a constant income tax-infrastructure subsidy combination is jointly required to attain the first-best equilibrium under private provision, the optimal income tax rate must be time-varying under government provision.

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1 Introduction

The recent policy shift in developing countries towards market provision of many goods and services traditionally viewed as being in the domain of "public goods" has once again put into question the role of government in economic progress. One such "public good" is infrastructure services, which include roads, power, water and sewerage, irrigation, transportation and communications. While privately provided infrastructure services are quite common in the developed world, their provision in the developing world is still perceived to be in the government's domain, though that perception is rapidly changing. The 1990's witnessed the first major shift from government to private provision, with 132 low- and middle-income countries transferring about 2,500 infrastructure projects to the private sector. A recent World Bank study estimates that between 1990-2002, private sector commitments to infrastructure in developing countries totalled about $805 billion, or $62 billion per year, which accounted for about 25 percent of total infrastructure spending (Estache, 2004).\(^1\) The privatization of infrastructure in the developing countries has mainly been driven by a rapid growth in demand and increasing public disenchantment with the performance and quality of state-provided services.\(^2\) At the same time, many governments have significantly reduced public-sector spending and borrowing following the debt and fiscal crises of the 1980s. Though infrastructure capital (often referred to as "public capital") has been regarded as an essential ingredient for growth and development, very little attention has been paid to the issue of its provision in the growth literature.\(^3\)

This paper studies the behavioral differences between economies where infrastructure is privately provided and those in which the government is the sole provider, and how these differences, in turn, determine the impact of fiscal policy on macroeconomic performance and welfare. In capturing these behavioral differences, we focus on two characteristics of public goods that are potentially important for their pricing and provision, namely rivalry and excludability; see Cornes and Sandler (1996). Excludability implies that the services from an underlying public good may not be available to all users, possibly due to a pricing mechanism (e.g. user fees and tolls). On the other hand, rivalry means that the services derived by an individual from a public good may be affected by the services derived by others. Though the initial literature on public investment and growth treated infrastructure as a "pure" public good which is non-rival and non-excludable, later works have

\(^1\)Estache (2004) points out that a major incentive for growing private participation in infrastructure provision is the high expected returns from investment. Canning and Bennathan (2000) and Briceno et al. (2004) estimate that in developing countries, the expected returns from investment in telecommunications are between 30-40 percent, while the corresponding returns in electricity generation and road construction are 40 and 80 percent, respectively.

\(^2\)One excellent example can be found in India which, till the early 1990's, was basically a closed economy with a huge public sector that operated on strong socialist principles. During the 1990's, as India embarked on an elaborate phase of liberalization, infrastructure provision and privatization became contentious issues. Recently, it was announced that the newly proposed interstate highway system will be privately built and operated, with tolls and user fees being the main instruments of financing. While the telecommunications sector has been largely privatized, the nation's airport system is also currently under privatization. Some states have privatized the provision of power and electricity as well. A recent survey of India's economic reforms can be found in Ahluwalia (2002).

\(^3\)The voluminous empirical literature on the productivity impact of infrastructure or "public capital" started with the findings of Aschauer (1989), and an early review can be found in Gramlich (1994).
indeed studied these properties, albeit to varying degrees. However, most of this literature assumes that infrastructure capital is provided directly by the government and the private sector merely takes its stock as exogenously given in making allocation decisions. It is therefore important to study the effects of excludability and rivalry when infrastructure is privately provided in comparison to when it is publicly provided.

The concept of excludability admits the existence of an implicit or explicit pricing system that can prevent universal access to a public good. In the context of infrastructure, excludability might arise through the imposition of "user fees" like tolls, taxes, or entrance fees. Ott and Turnovsky (2006) discuss examples of different types of highway toll systems (time based and distance based) prevalent in the European Union. However, the existence of a pricing mechanism also raises the possibility of market provision of infrastructure, and the case for government provision is thereby weakened. On the other hand, when private providers are responsible for financing and pricing infrastructure capital, it is important to understand the determinants of such a pricing structure. We argue that the market price of infrastructure must be linked not only to its own usage, but to the usage of private capital as well. We formalize the concept of usage by introducing endogenous utilization decisions for both private capital and infrastructure, and linking them to the corresponding depreciation rates. This innovation turns out to be critically important, as the internalization of these utilization decisions varies according to the mode of provision of infrastructure.

To compare our analysis to the existing literature, we assume that while government-provided infrastructure capital is non-excludable, the corresponding services provided by the private sector are excludable. Therefore, under private provision, a user (in our case, the representative agent) can internalize the effects of utilization of both private and infrastructure capital on the production of output, and therefore has the flexibility to adjust resource allocation along these margins in response to a fiscal shock. It turns out that the utilization rates of the two types of capital are interdependent along the equilibrium path and jointly determine their respective market prices. In contrast, when the government provides the entire stock of infrastructure, the underlying services are non-excludable and, consequently, treated as exogenously given by the private agent. The agent, therefore, does not internalize the effect of its allocation decisions on the accumulation.

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4 The formal treatment of public investment in an intertemporal framework can be traced to Arrow and Kurz (1970) in the context of the neoclassical growth model. Though Barro (1990) revived this discussion in the context of endogenous growth, these early papers assumed non-rivalry in modeling public investment. Later contributions, however, have incorporated rivalry in growth models, mainly in the form of congestion; See, for example, Turnovsky (1996), Glomm and Ravikumar (1997), and Fisher and Turnovsky (1998). Excludability has been studied extensively in the public economics literature, as in Brito and Oakland (1980), Burns and Walsh (1981) and Fraser (1996). However, it has received very limited attention in the growth literature, though a recent contribution can be found in Ott and Turnovsky (2006).

5 See, for example, Baxter and King (1993), Futagami et al. (1993), and Rioja (2003).

6 The concept of capital utilization refers to the intensity or frequency with which capital equipment is operated, and is a popular construct in the business cycle literature; see Keynes (1936) for a very early discussion, and Lucas (1970), Calvo (1975), and Greenwood et al. (1988) for more recent contributions. In the context of intertemporal growth, its use is less prevalent, though some recent studies by Imbs (1999), Dalgaard (2003), and Chatterjee (2005) have demonstrated its importance for the dynamics of growth and convergence. However, to our knowledge, there is no known study of capital utilization in models of public investment and growth.
utilization, and depreciation of the government-provided infrastructure capital. This behavioral difference between the two regimes leads to critical differences in their responses to fiscal policy shocks, which eventually translate into substantial differences in welfare gains or losses.

The accumulation and usage of infrastructure also generates externalities that lead to some form of rivalry. Congestion is a classic example of rivalry and has been extensively studied in the growth literature (see footnote 3). We extend the notion of rivalry to introduce a production externality that can either reinforce or offset the effects of congestion. Specifically, we assume that the economy-wide ratio of infrastructure to private capital generates an aggregate production externality for the private agent. For example, given the stock of private capital, an increase in the stock of infrastructure mitigates the effects of congestion (e.g., given the number of cars, if the number of lanes in a highway is increased) and has a positive impact on aggregate productivity. Conversely, given the stock of infrastructure, if the stock of private capital increases, then the effects of congestion are enhanced, with a dampening effect on productivity. The congestion and production externality parameters are, therefore, important determinants of the market prices of the two capital stocks and their respective utilization rates.

Our paper is related to a small literature that has studied the provision of infrastructure or "public capital" in the context of growth. The first mention of the possibility of private provision can be found in Glomm and Ravikumar (1997), though they do not provide any formal treatment. A formal analysis, however, is provided by Devarajan et al. (1998), who examine the choice between private and public provision by evaluating the distortions created by the tax system in financing either direct government provision or subsidies to private providers. More recently, Chatterjee (2006) generalizes the Devarajan et al. framework to the open economy and shows that the choice between private and government provision depends crucially on the economy’s structural parameters such as the elasticity of substitution in production and the size of externalities, as well as borrowing constraints in international capital markets.

We distinguish our approach from the previous literature by focusing on certain aspects of infrastructure provision absent from previous analyses. First, we focus on behavioral differences between the regimes of private and government provision. Second, we examine excludability and pricing of infrastructure by explicitly introducing endogenous utilization decisions. The inherent differences in the extent to which these decisions are internalized across the two regimes eventually end up determining their welfare responses to policy shocks. Third, the literature on private provision does not examine the consequences of rivalry, whereas we introduce both congestion and an aggregate production externality that significantly affect utilization rates and market prices. Fourth, we adopt a more flexible budget constraint for the government by introducing lump-sum taxes and debt financing. This aspect is absent both in Devarajan et al. (1998) and Chatterjee (2006), who focus primarily on the distortions created by the income tax as the government balances its budget. By allowing for non-distortionary sources of financing, we can decouple the effects of spending and revenue generation, making the comparison between the two regimes more
transparent. This feature of the model also enables us to address another important issue absent from the previous literature: how private and government provision of infrastructure affect the design of optimal fiscal policy.7

Our results indicate that if the government wants to stimulate infrastructure investment, a targeted subsidy to private providers yields significantly higher welfare gains than if the government were to directly provide the additional investment without any private involvement. This result is robust to both the underlying financing instrument (lump-sum versus distortionary taxation) and variations in the congestion and production externality parameters. Therefore, asymmetric tax distortions (as in Devarajan et al., 1998) or structural conditions (as in Chatterjee, 2006) are not essential in comparing the two modes of infrastructure provision. Rather, the inherent differences in the extent to which excludability (or the lack of it) permits the internalization of utilization decisions and market prices form the crux of our explanation. This represents a significant departure from earlier analysis, which assumed that if the government has non-distortionary financing instruments at its disposal, the choice between public and private provision is irrelevant. We also find that an income tax is more distortionary in a privatized economy than under government provision, because an increase in the income tax rate reduces the market return to both private and infrastructure capital and causes adjustments in both capital stocks as well as their utilization rates. However, under government provision, the agent takes infrastructure as exogenously given and therefore has only one margin of adjustment, which is private capital and its utilization. Again, the introduction of endogenous capital utilization leads to a sharp contrast with Devarajan et al. (1998), who find that tax distortions are higher under government provision. As for the design of optimal fiscal policy, we show that under private provision, the government needs both an income tax and infrastructure subsidy to attain the first best optimum. By contrast, under direct government provision, the burden of attaining optimality falls solely on a time-varying income tax rate, because under government provision, the market price of infrastructure and its utilization rate are not internalized by the private agent, and thus a time-varying tax rate must be imposed to track the dynamic adjustment of these variables.

2 Analytical Framework

We consider $N$ identical and infinitely lived representative agents, who maximize utility from consumption according to

$$U = \int_0^\infty \frac{C^\gamma}{\gamma} e^{-\beta t} dt, \quad -\infty < \gamma \leq 1$$

(1)

Each agent also produces output ($Y$) using its individual stocks of private capital ($K$) and infrastructure capital ($K_g$). The accumulation of private capital, defined as an amalgam of physical

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7There is no discussion of optimal fiscal policy in earlier studies on this issue, such as Devarajan et al. (1998) and Chatterjee (2006). In these papers, the absence of lump-sum taxes or other non-distortionary financing instruments in the government’s budget constraint generates time-inconsistency in implementing optimal fiscal policy.
and human capital, is undertaken by the agent, while infrastructure capital may be provided either "privately" (by the representative agent, in which case it would be excludable) or "publicly" (by the government, where it would be non-excludable). The production function can be described as follows

\[ Y = A \left( \frac{\bar{K}_g}{\bar{K}} \right)^{\varepsilon} (u_k K)^{\eta} (K^s_g)^{1-\eta}, \quad 0 < \eta < 1, \quad 0 < \varepsilon < 1, \quad 0 < \varepsilon + \eta < 1 \] (2)

where \( \bar{K} \) and \( \bar{K}_g \) represent the economy-wide aggregate stocks of private capital and infrastructure, respectively, while \( K \) and \( K_g \) denote the corresponding stocks available to an individual agent.

The accumulation and usage of the two factors of production generate two sources of externalities for the private sector, which the individual agent cannot internalize. The first is a congestion externality, which impacts on the services derived from infrastructure. Defining \( u_k \) and \( u_g \) as the respective rates of utilization of private capital and infrastructure, we specify that the services derived from the agent’s stock of infrastructure capital, \( K^s_g \), are proportional to (i) the usage of the privately owned stock of infrastructure \( (u_g K_g) \), and (ii) the usage of the agent’s individual stock of private capital relative to its aggregate stock:\(^8\)

\[ K^s_g = u_g K_g \left( \frac{u_k K}{\bar{K}} \right)^{1-\sigma}, \quad 0 \leq \sigma \leq 1 \] (2a)

where \( \sigma \) measures the degree of relative congestion. Additionally, given the aggregate stock of utilized private capital, the accumulation of the economy-wide stock of infrastructure helps mitigate the effects of congestion, thereby generating an aggregate production externality, captured by \( \varepsilon \) in (2).\(^9\)

The aggregate utilized stocks of private and infrastructure capital are defined as

\[ \bar{K} = N(u_K K) \] (2b)
\[ \bar{K}_g = N(u_g K_g) \] (2c)

The rates of accumulation of each type of capital are given by

\[ \dot{K} = I - \delta_k(u_k) K \] (3a)
\[ \dot{K}_g = G - \delta_g(u_g) K_g \] (3b)

\(^8\)We define the rate of utilization (or usage) of a given type of capital stock (private or infrastructure) as the speed or intensity with which it is operated (for example, "workweek," "hours per day," etc.), as in Taubman and Wilkinson (1970) and Calvo (1975).

\(^9\)The aggregate production externality captures two offsetting aspects of the factor accumulation and utilization process on the economy’s capacity to produce output. For example, given a stock of roads and highways, the number of cars driven on them generates congestion and reduces the aggregate output elasticity of private capital. On the other hand, given the usage of private cars, more roads might reduce the effects of congestion and enhance the economy’s productive capacity. For an example of a similar aggregate production externality in the context of foreign aid and growth, see Chatterjee and Turnovsky (2006).
where $I$ and $G$ measure the flow of new investment into the two capital goods, respectively, and $\delta_k$ and $\delta_g$ denote their corresponding depreciation rates. A critical point to note here is that the rates of depreciation of each capital good depend on their respective rates of utilization:

$$\delta_i(u_i) = \frac{u_i^{\phi_i}}{\phi_i}, \quad \phi_i > 1, \quad i = k, g$$  \hspace{1cm} (4)

The parameters $\phi_i (i = k, g)$ in (4) measure the elasticity of depreciation with respect to utilization of the underlying stock of capital.$^{10}$ Finally, we assume that the accumulation of both types of capital is costly and involves convex costs of adjustment

$$\Gamma (I, K) = I \left[ 1 + \frac{h_1}{2} \frac{I}{K} \right]$$  \hspace{1cm} (5a)

$$\Omega (G, K_g) = G \left[ 1 + \frac{h_2}{2} \frac{G}{K_g} \right]$$  \hspace{1cm} (5b)

The budget constraints for the representative agent and the government will depend on how infrastructure is provided in an economy, i.e., whether it is provided by the (i) private representative agent, (ii) government, or (iii) a social planner (in this case both capital goods are provided by the planner). The sections below describe each regime of infrastructure provision.

3 Private Provision of Infrastructure

Under this regime, the representative agent provides both private and infrastructure capital and chooses their respective rates of utilization. As a result, infrastructure, just like private capital, is an excludable good. However, the agents’ failure to internalize the externalities associated with the two types of capital provides an incentive for government intervention. Such an intervention can take place through a wide array of fiscal instruments, namely the income tax ($\tau_y$), a tax on interest income ($\tau_b$) generated by the issue of government bonds ($b$), a lump-sum tax ($T$), and a subsidy targeted for infrastructure investment ($s$). The private agent’s flow budget constraint is given by

$$\dot{b} = (1 - \tau_b) rb + (1 - \tau_y) Y - C - \Gamma (I, K) - (1 - s) \Omega (G, K_g) - T$$  \hspace{1cm} (6a)

The government finances any excess of expenditures over tax revenues by issuing debt in the form of infinitesimally short government bonds. The evolution of government debt is described by

$$\dot{b} = (1 - \tau_b) rb + s \Omega (G, K_g) - \tau_y Y - T$$  \hspace{1cm} (6b)

$^{10}\phi_i = u_i \delta'(u_i)/\delta(u_i), \quad i = k, g$. Under this specification, the marginal depreciation cost of utilization of a capital stock, $\delta'(u_i)$, is variable. Note that as $\phi_i \to \infty$, $\delta_i(u_i) \to 0$ and $u_i \to 1$. The conventional assumption in the growth literature is that of a constant depreciation rate, so that $\delta_i(u_i) = 0$ and $u_i = 1$. Equation (4) represents the "depreciation-in-use" function, which is a standard specification in many Real Business Cycle models; see Burnside and Eichenbaum (1996).
Finally, the relevant production function for the representative agent is

\[ Y = A \left( \frac{\bar{K}_g}{K} \right)^\varepsilon (u_KK)^{1-\sigma(1-\eta)} (u_gK_g)^{1-\eta} (\bar{K})^{(\eta-1)(1-\sigma)} \]  

(7)

Combining (6a) and (6b) yields the economy’s aggregate resource constraint

\[ Y = C + \Gamma (I, K) + \Omega (G, K_g) \]  

(8)

The representative agent maximizes (1), subject to (3a), (3b), (6a), and (7), while taking note of (4), (5a) and (5b). It is important to emphasize here that though the aggregate relationships (2b) and (2c) are not internalized by the agent in performing its optimization, they hold in equilibrium. Also, in deriving the equilibrium conditions, we have normalized \( N = 1 \), without loss of generality. The optimality conditions are

\[ C^{\gamma-1} = \lambda \]  

(9a)

\[ A(1 - \tau_g)[1 - \sigma(1 - \eta)] \left( \frac{u_gK_g}{K} \right)^{1-(\eta-\varepsilon)} \frac{u_K^{\eta-\varepsilon-1}q_k^{\eta-\varepsilon}}{q_k} = u_k^{\phi_k-1} \]  

(9b)

\[ A(1 - \tau_g)(1 - \eta) \left( \frac{u_kK_g}{K_g} \right)^{\eta-\varepsilon} \frac{u_g^{\varepsilon-\eta}q_g^{\varepsilon-\eta}}{q_g} = u_g^{\phi_g-1} \]  

(9c)

\[ i = \frac{I}{Y} = \left( \frac{q_k - 1}{h_1} \right) \left( \frac{K}{Y} \right) \]  

(9d)

\[ g = \frac{G}{Y} = \left( \frac{q_g - 1}{h_2} \right) \left( \frac{K_g}{Y} \right) \]  

(9e)

\[ \beta - \frac{\dot{\lambda}}{\lambda} = (1 - \tau_b)r \]  

(9f)

\[ \frac{\dot{q}_k}{q_k} + \frac{A(1 - \tau_g)[1 - \sigma(1 - \eta)]u_k^{\eta-\varepsilon}u_g^{1-(\eta-\varepsilon)}(K_g/K)^{1-(\eta-\varepsilon)}}{q_k} + \frac{(q_k - 1)^2}{2h_1q_k} - \delta_k(u_k) = (1 - \tau_b)r \]  

(9g)

\[ \frac{\dot{q}_g}{q_g} + \frac{A(1 - \tau_g)(1 - \eta)u_k^{\eta-\varepsilon}u_g^{1-(\eta-\varepsilon)}(K_g/K)^{\varepsilon-\eta}}{q_g} + \frac{(q_g + s - 1)^2}{2(1 - s)h_2q_g} - \delta_g(u_g) = (1 - \tau_b)r \]  

(9h)

The above optimality conditions can be interpreted as follows. (9a) equates the marginal utility from consumption to that of private wealth, measured by the shadow price \( \lambda \). Equations (9b) and (9c) represent the optimal decisions regarding the utilization of the two capital stocks, respectively.
(9b) equates the after-tax marginal benefit from utilizing private capital, valued by its shadow price, to its marginal depreciation cost. Similarly, (9c) equates the after-tax marginal benefit and cost of utilizing infrastructure. Two things must be noted here. First, the factor utilization decisions do not internalize the congestion and production externalities (σ and ε), and hence are sub-optimal. Second, given that infrastructure has certain public-good characteristics, (9c) is also a formal statement of exclusion, showing that the usage of infrastructure depends, among other things, on its shadow price, \( q_g \). (9d) and (9e) describe the allocation of output to investment in the two capital goods, respectively. These allocations depend (i) positively on the respective shadow prices and (ii) inversely on the respective average products. As a result, the fraction of output allocated to either capital good is time-varying along the transition path to the steady-state equilibrium. Equations (9f)-(9h) represent no-arbitrage conditions for consumption, private capital and infrastructure respectively, thereby ensuring an interior equilibrium allocation. Equations (9g) and (9h) also describe the evolution of the shadow (market) price of each capital good, which is crucial for clearing their respective markets.

The optimality conditions can also be used to derive the equilibrium growth rates for private capital, infrastructure, and consumption, respectively:

\[
\Psi_k = \frac{\dot{K}}{K} = \frac{q_k - 1}{h_1} - \delta_k(u_k)
\]

\[
\Psi_g = \frac{\dot{K}_g}{K_g} = \frac{q_g + s - 1}{(1 - s)h_2} - \delta_g(u_g)
\]

\[
\Psi_c = \frac{\dot{C}}{C} = \frac{(1 - \tau_b)r - \beta}{1 - \gamma}
\]

Note that the growth rates of both private capital and infrastructure depend on their respective utilization rates. Moreover, (10b) clearly illustrates the dual role played by the subsidy in influencing the evolution of the privately provided stock of infrastructure: it increases the shadow price and lowers the cost of investment, thereby encouraging its accumulation.

### 3.1 Macroeconomic Equilibrium

The presence of both private capital and infrastructure implies that the equilibrium path will be characterized by transitional dynamics. Therefore, we will describe the macroeconomic equilibrium in terms of the shadow prices \( q_k \) and \( q_g \) and the following stationary variables: \( z = K_g/K \), the ratio of infrastructure to private capital, and \( c = C/K \), the consumption-private capital ratio. The first step in deriving the macroeconomic equilibrium is the determination of the equilibrium utilization rates for private capital and infrastructure. These can be obtained by solving the static equilibrium

\[11\]The shadow prices \( q_k \) and \( q_g \) are measured in terms of the (unitary) price of government bonds. Consequently, the shadow value of wealth, \( \lambda \), is used as a numeraire.
conditions (9b) and (9c):

\[ u_k \equiv u_k(q_k, q_g, z) = \left[ \Delta_1 \frac{z^{(\phi_g - 1)(1-\alpha)}}{q_k^\alpha q_g^{1-\alpha}} \right]^{\frac{1}{\zeta}} \]  \tag{11a}

\[ u_g \equiv u_g(q_k, q_g, z) = \left[ \Delta_2 \frac{z^{\alpha(1-\phi)}}{q_k^{\alpha} q_g^{k-\alpha}} \right]^{\frac{1}{\zeta}} \]  \tag{11b}

where, \( \alpha = \eta - \varepsilon, \zeta = -[\alpha \phi_g + (1 - \alpha) \phi_k - \phi_k \phi_g], \Delta_1 = [1 - \sigma(1 - \eta)]^{\phi_g + (1-\alpha)(1-\eta)^{1-\alpha}} [A(1 - \tau)\phi_g], \)

\( \text{and} \Delta_2 = [1 - \sigma(1 - \eta)]^{\alpha} (1 - \eta)^{\phi_k - \alpha} [A(1 - \tau y)]^\phi_k. \) Note that \( \alpha \) represents the aggregate output elasticity of private capital (after considering its external effects on productivity).

The utilization rates for the two capital stocks depend on shadow prices, the infrastructure-private capital ratio, and on the structural, policy, and externality parameters of the model. To get some intuition on the behavior of the utilization rates, consider the following partial derivatives, under the mild restriction that \( \zeta > 0 \):

\[ \frac{\partial u_k}{\partial z} = \frac{(\phi_g - 1)(1-\alpha)}{\zeta} \left( \frac{u_k}{z} \right) > 0, \quad \frac{\partial u_g}{\partial z} = \frac{\alpha(1 - \phi_k)}{\zeta} \left( \frac{u_g}{z} \right) < 0 \]

\[ \frac{\partial u_k}{\partial q_k} = \frac{(1 - \phi_g - \alpha)}{\zeta} \left( \frac{u_k}{q_k} \right) < 0, \quad \frac{\partial u_g}{\partial q_k} = -\frac{\alpha}{\zeta} \left( \frac{u_g}{q_k} \right) < 0 \]

\[ \frac{\partial u_k}{\partial q_g} = \frac{(\alpha - 1)}{\zeta} \left( \frac{u_k}{q_g} \right) < 0, \quad \frac{\partial u_g}{\partial q_g} = \frac{(\alpha - \phi_k)}{\zeta} \left( \frac{u_g}{q_g} \right) < 0 \]

Intuitively, an increase in the proportion of infrastructure relative to private capital enhances the marginal product of private capital (being complementary inputs in production), thereby increasing its rate of utilization. On the other hand, in the presence of diminishing returns, an increase in infrastructure reduces its own average and marginal product, leading to a decline in its own rate of utilization. The respective utilization rates are also negatively related to the shadow (market) prices of the two capital goods, indicating that an increase in price makes investment costly, thereby reducing the rates of utilization. Further, (11a) and (11b) immediately determine the equilibrium depreciation rates of each capital good, as well as their evolution over time.

The core equilibrium dynamics can be expressed as

\[ \frac{\dot{z}}{z} = \frac{\dot{K}_g}{K_g} - \frac{\dot{K}}{K} = \left[ \frac{q_g + s - 1}{(1-s)h_2} - \delta_g(u_g) \right] - \left[ \frac{q_k - 1}{h_1} - \delta_k(u_k) \right] \]  \tag{12a}

\[ \dot{q}_k = (1 - \tau_b) r q_k - A(1 - \tau_y) [1 - \sigma(1 - \eta)] u_k^\alpha u_g^{1-\alpha} z^{1-\alpha} - \frac{(q_k - 1)^2}{2 h_1} + \delta_k(u_k) q_k \]  \tag{12b}

\[ \dot{q}_g = (1 - \tau_b) r q_g - A(1 - \tau_y) (1 - \eta) u_k^\alpha u_g^{1-\alpha} z^{-\alpha} - \frac{(q_g + s - 1)^2}{2(1-s)h_2} + \delta_g(u_g) q_g \]  \tag{12c}
where \( u_k \) and \( u_g \) are given in (11). The evolution of the consumption-private capital ratio is independent of the core dynamics and is given by

\[
\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left[ \frac{(1 - \tau_b)\alpha - \beta}{1 - \gamma} \right] - \left[ \frac{q_k - 1}{h_1} - \delta_k(u_k) \right] \tag{12d}
\]

The steady-state equilibrium is characterized by balanced growth and is attained when \( \dot{z} = \dot{q}_k = \dot{q}_g = \dot{c} = 0:

\[
\frac{\tilde{q}_g + s - 1}{(1 - s)h_2} - \delta_g(\tilde{u}_g) = \frac{\tilde{q}_k - 1}{h_1} - \delta_k(\tilde{u}_k) \tag{13a}
\]

\[
A(1 - \tau_y)\left[ 1 - \sigma(1 - \eta) \right] \tilde{q}_k^\alpha \tilde{u}_g^{1 - \alpha} \tilde{z}^{1 - \alpha} + \frac{(\tilde{q}_k - 1)^2}{2h_1\tilde{q}_k} - \delta_k(\tilde{u}_k) = (1 - \tau_b)\tilde{r} \tag{13b}
\]

\[
\frac{A(1 - \tau_y)(1 - \eta)\tilde{u}_k^\alpha \tilde{u}_g^{1 - \alpha} \tilde{z}^{-\alpha}}{\tilde{q}_g} + \frac{(\tilde{q}_g + s - 1)^2}{2(1 - s)h_2\tilde{q}_g} - \delta_g(\tilde{u}_g) = (1 - \tau_b)\tilde{r} \tag{13c}
\]

\[
\frac{(1 - \tau_b)\tilde{r} - \beta}{1 - \gamma} = \frac{\tilde{q}_k - 1}{h_1} - \delta_k(\tilde{u}_k) \tag{13d}
\]

In addition, the aggregate resource constraint in steady state is given by

\[
A\tilde{q}_k^\alpha \tilde{u}_g^{1 - \alpha} \tilde{z}^{1 - \alpha} = \tilde{c} + \frac{\tilde{q}_g^2 - 1}{2h_1} + \left[ \frac{\tilde{q}_g^2 - (1 - s)^2}{2(1 - s)h_2} \right] \tilde{z} \tag{13e}
\]

Using (11a) and (11b) in (13a)-(13d), we can solve for the steady-state values of \( \tilde{z}, \tilde{r}, \tilde{q}_k, \) and \( \tilde{q}_g \). Substituting these values in the aggregate resource constraint (13e) immediately yields the steady-state level of the consumption-capital ratio, \( \tilde{c} \).

### 3.2 Optimal Fiscal Policy

In a decentralized economy with private provision of all factors of production and externalities, maximizing economic welfare might be an important objective for the government. This optimal policy intervention is attained by choosing the appropriate set of tax and subsidy (expenditure) rates \( (\tau_y, \tau_b, \tilde{s}) \) that ensures an equilibrium allocation that replicates a centrally planned economy.

The central planner’s allocation is always the "first-best," since all externalities are internalized ex-ante. To characterize optimal fiscal policy in the decentralized economy, we will first describe the central planner’s equilibrium resource allocation.

The relevant production function for the central planner is given by

\[
Y = A(u_k K)^{\eta - \varepsilon} (u_g K_g)^{1 - (\eta - \varepsilon)} = A(u_k K)^\alpha (u_g K_g)^{1 - \alpha} \tag{14}
\]

\[\text{12} \text{The linearized dynamics corresponding to this steady state can be described as } \dot{X} = \Lambda \left( X - \bar{X} \right), \text{ where } \bar{X}' = (z, q_k, q_g), \bar{X}' = (\tilde{z}, \tilde{q}_k, \tilde{q}_g), \text{ and } \Lambda \text{ represents the 3x3 coefficient matrix of the linearized system. It can be verified that the equilibrium is a saddle path with one stable (negative) and two unstable (positive) eigenvalues.} \]
Note that in (14), the centrally planned economy is not subject to congestion, since the planner takes into account (2b) and (2c). Moreover, the effect of the aggregate production externality ($\varepsilon$) is also internalized by the planner. The central planner maximizes (1), subject to the aggregate resource constraint (8) and the accumulation equations (3a) and (3b), given the production function in (14). Denoting all equilibrium variables in the centrally planned economy with a superscript "$^*$", the optimal capital utilization rates are given by

$$u_k^* = \left[ \Delta_1^* \left( \frac{(z^*)^{(\phi_y-1)(1-\alpha)}}{(q^*_k)^{\phi_y+\alpha-1}(q^*_g)^{1-\alpha}} \right) \right]^{\frac{1}{\gamma}} \tag{15a}$$

$$u_g^* = \left[ \Delta_2^* \left( \frac{(z^*)^{\alpha(1-\phi_k)}}{(q^*_k)^{\alpha}(q^*_g)^{\alpha-\alpha}} \right) \right]^{\frac{1}{\gamma}} \tag{15b}$$

where $\alpha = \eta - \varepsilon$, $\Delta_1^* = \alpha^{\phi_y+\alpha-1}(1-\alpha)^{1-\alpha}A^{\phi_y}$, and $\Delta_2^* = \alpha^{\alpha}(1-\alpha)^{\phi_k-\alpha}A^{\phi_y}$.

The corresponding steady-state conditions for a centrally planned economy are

$$\frac{\tilde{q}_g^* - 1}{h_2} - \delta_g(\tilde{u}_g^*) = \frac{\tilde{q}_k^* - 1}{h_1} - \delta_k(\tilde{u}_k^*) \tag{16a}$$

$$\frac{(1 + \tilde{v}\tilde{g})}{\hat{g}} A(\tilde{u}_k^*)^{\alpha} (\tilde{u}_g^*)^{1-\alpha} (\tilde{z}^*)^{1-\alpha} \frac{(\tilde{q}_k^* - 1)^2}{2h_1\tilde{q}_k^*} - \delta_k(\tilde{u}_k^*) = \tilde{r}^* \tag{16b}$$

$$\frac{(1 + \tilde{v}\tilde{g})}{\hat{g}} (1 - \alpha) A(\tilde{u}_k^*)^{\alpha} (\tilde{u}_g^*)^{1-\alpha} (\tilde{z}^*)^{-\alpha} \frac{(\tilde{q}_g^* - 1)^2}{2h_2\tilde{q}_g^*} - \delta_g(\tilde{u}_g^*) = \tilde{r}^* \tag{16c}$$

$$\frac{\tilde{r}^* - \beta}{1 - \gamma} = \frac{\tilde{q}_k^* - 1}{h_1} - \delta_k(\tilde{u}_k^*) \tag{16d}$$

In (16b) and (16c), $\tilde{g}$ represents the fraction of output allocated to infrastructure investment by the central planner, and $\tilde{v}$ denotes the corresponding shadow value (resource cost) of this allocation. In the case where the central planner chooses this fraction endogenously (optimally), $\tilde{v} = 0$.

To derive optimal fiscal policy in the decentralized economy, we compare the steady-state relationships (13a)-(13d), with the corresponding relationships in the centrally planned economy, (16a)-(16d), assuming that the central planner sets $\tilde{g}$ optimally (so that $\tilde{v} = 0$ in (16b) and (16c)). This enables us to solve for the optimal rates for the fiscal instruments in the decentralized economy:

$$\tilde{\tau}_b = 0 \tag{17a}$$

$$\tilde{\tau}_y = \frac{\varepsilon + (1 - \eta)(1 - \sigma)}{1 - \sigma(1 - \eta)} \tag{17b}$$

$$\tilde{s} = \frac{1}{1 + \omega} \tag{17c}$$
where
\[ \omega = \frac{\tilde{q}_g^2}{2h_2A(\tilde{u}_k/\tilde{z}_g)^{\eta-\epsilon}(\epsilon + (1-\eta)\tilde{\tau}_y)} \]

The optimal tax on bond income, given by (17a), must be zero as there are no capital market imperfections. The optimal tax on income, \( \tilde{\tau}_y \), given in (17b), takes into account the effect of both the congestion and the aggregate production externality. To see this, consider the case where there is no congestion, but the aggregate production externality is positive, i.e., \( \sigma = 1 \) and \( \varepsilon > 0 \). In that case,
\[
\tilde{\tau}_y = \frac{\varepsilon}{\eta} \quad (17a')
\]

(17a’) implies that even without congestion, the optimal income tax rate must be positive. The intuition behind this result is that the actual output elasticity of private capital is \( (\eta - \varepsilon) \), but the private agent internalizes only \( \eta \). Therefore, the optimal tax rate on capital income enables the agent to internalize the negative effect of \( \varepsilon \), while adjusting it by \( \eta \), which has already been internalized ex-ante. When \( \varepsilon = 0 \), but congestion is present \( (0 < \sigma < 1) \), we get the result familiar from much of the existing literature; see Turnovsky (1997).

The optimal tax rates described in (17a) and (17b) are, however, insufficient to attain the first-best resource allocation, because both externalities affect the usage and accumulation of infrastructure as well, which in turn requires an additional corrective fiscal instrument. This is given by the optimal infrastructure subsidy in (17c), which takes into account not only the aggregate production externality \( \varepsilon \), but also the equilibrium factor utilization rates and the shadow price of infrastructure, all of which crucially determine its services. Further, though the government has access to non-distortionary sources of financing, the optimal infrastructure subsidy must be partially financed by income tax revenues. This makes intuitive sense, since an income tax affects the returns from both factors of production. More simply, even though the optimal income tax rate is designed to control for the externalities generated by private capital accumulation, it also affects the marginal return and usage of infrastructure capital. Therefore, a part of the tax revenues collected by the government are rebated back to the private sector through the infrastructure subsidy.

4 Government Provision of Infrastructure

We now consider a decentralized economy where the entire stock of infrastructure is provided by the government. Consequently, infrastructure services are non-excludable and the private agent takes its stock as exogenously given in making allocation decisions. However, production is still subject to the two externalities and is given by
\[
Y = A(u_K K)^{1-\sigma(1-\eta)}(\tilde{u}_g K_g)^{1-\eta+\epsilon}(\tilde{K})(\eta-1)(1-\sigma)-\varepsilon \quad (18)
\]

Note that, since the government directly provides infrastructure capital, we set \( \tilde{K}_g = \tilde{u}_g K_g \) at the outset. Also, since the agent treats \( K_g \) as exogenously given, it does not internalize the
utilization rate, $u_g$, and its effect on depreciation. As a result, the agent treats infrastructure depreciation as an exogenous constant, $\delta_g = \delta_g$, for all $t$. The immediate consequence of this is that the marginal cost of infrastructure utilization is zero for the private agent. This implies that the rate of infrastructure utilization will also be treated by the private agent as an exogenous constant, i.e., $u_g = \bar{u}_g$, for all $t$ ($0 < \bar{u}_g \leq 1$). This is the key behavioral difference between this regime and the privatized one described in section 3.

The corresponding private flow budget constraint is given by

$$\dot{b} = (1 - \tau_b)rb + (1 - \tau_y)Y - C - \Gamma(I, K) - T \quad (19)$$

The government’s budget constraint is now given by

$$\dot{b} = (1 - \tau_b)rb + \Omega(G, K_g) - \tau_y Y - T \quad (19b)$$

Note that since the government provides infrastructure capital, its installation costs do not enter (19b). The aggregate resource constraint continues to be given by (8).

The private agent maximizes (1), subject to (19a) and (3a), given (4a), (5a), and (18). As in section 3, we have normalized $N = 1$ and set $K = u_kK$ (ex-post) in deriving the equilibrium conditions:

$$C^{\gamma - 1} = \lambda \quad (20a)$$

$$\beta - \frac{\dot{\lambda}}{\lambda} = (1 - \tau_b)r \quad (20b)$$

$$A(1 - \tau_y)[1 - \sigma(1 - \eta)] \left( \frac{\bar{u}_gK_g}{K} \right)^{1-(\eta - \epsilon)} \frac{u_k^{\eta - \epsilon - 1}}{q_k} = u_k^{\phi_k - 1} \quad (20c)$$

$$i \equiv \frac{I}{K} = \frac{q_k - 1}{h_1} \quad (20d)$$

$$\frac{\dot{q}_k}{q_k} + \frac{A(1 - \tau_y)[1 - \sigma(1 - \eta)] u_k^{\eta - \epsilon}(\bar{u}_gK_g/K)^{1-(\eta - \epsilon)}}{q_k} + \frac{(q_k - 1)^2}{2h_1q_k} - \delta_k(u_k) = (1 - \tau_b)r \quad (20e)$$

The interpretation of the optimality conditions (20a)-(20e) is analogous to those in section 3. However, there are some key differences in the structure of the equilibrium. First, since the infrastructure utilization rate is exogenous to the private agent, a condition analogous to (9c) is now absent. Second, the evolution of the shadow price of infrastructure ($q_g$) is not internalized by the private agent and therefore is not part of the macroeconomic equilibrium.
The evolution of the government-provided stock of infrastructure capital is given by (3b). To maintain an equilibrium of sustained growth, the government must spend a fixed fraction of aggregate output on infrastructure investment:

\[ \dot{K}_g = G - \delta_g K_g, \quad G = gY, \quad 0 < g < 1 \] (3b')

where \( g \) is government spending on infrastructure investment, as a fraction of aggregate output. Given the government’s budget constraint in (19b), this expenditure can be financed through a variety of policy instruments, such as income and lump-sum taxes, as well as government debt. Recalling (9e), an interesting difference between the private and government provision model is that while \( g \) is constrained to be a constant fraction under government provision, it is time-varying under private provision, evolving with the shadow price of infrastructure and its average product along the transition path to the steady-state equilibrium.

4.1 Macroeconomic Equilibrium

The basic structure of the macro-dynamic equilibrium remains similar to section 3. However, the equilibrium is now described in terms of \( z, c \), and \( q_k \) only, and is independent of the shadow price of infrastructure, \( q_g \). The rate of utilization of private capital can be derived from (19c):

\[ u_k \equiv u_k(q_k, z) = \left[ \frac{A\{1 - \sigma(1 - \eta)\}(1 - \tau_y)(\bar{u}_g z)^{1-\alpha}}{q_k} \right]^{\frac{1}{1-\alpha}} \] (21)

Comparing (21) with its counterpart (11a) under private provision, we see that the choice of private capital utilization in this regime depends only on the shadow price of private capital and the ratio of infrastructure to private capital, but is independent of the shadow price of infrastructure as well as its utilization rate (since \( \bar{u}_g \) is a constant).

The core equilibrium dynamics are given by

\[ \frac{\dot{z}}{z} \equiv \frac{\dot{K}_g}{K_g} - \frac{\dot{K}}{K} = gAu_k^\alpha \bar{u}_g^{1-\alpha} z^{-\alpha} - \frac{q_k - 1}{h_1} - \delta_k(u_k) \] (22a)

\[ \frac{\dot{q}_k}{q_k} = (1 - \tau_b)r - \frac{A(1 - \tau_y)[1 - \sigma(1 - \eta)]u_k^\alpha \bar{u}_g^{1-\alpha} z^{1-\alpha}}{q_k} - \frac{(q_k - 1)^2}{2h_1q_k} + \delta_k(u_k) \] (22b)

where \( u_k \) is given by (21) and \( \alpha = \eta - \varepsilon \), as before. The evolution of the consumption-private capital ratio is given by

\[ \frac{\dot{c}}{c} \equiv \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left[ \frac{(1 - \tau_b)r - \beta}{1 - \gamma} \right] - \frac{q_k - 1}{h_1} - \delta_k(u_k) \] (22c)

The economy will attain its balanced growth steady-state equilibrium when \( \dot{z} = \dot{c} = \dot{q}_k = 0 \).
The corresponding steady-state conditions are

\[ gA\tilde{u}_k^\alpha \tilde{u}_g^{1-\alpha} \tilde{z}^{-\alpha} - \tilde{\sigma}_g = \frac{\tilde{q}_k - 1}{h_1} - \delta_k(\tilde{u}_k) \]  
\[ (1 - \tau_b)\tilde{r} - \beta = \frac{\tilde{q}_k - 1}{h_1} - \delta_k(\tilde{u}_k) \]  
\[ A(1 - \tau_y)\frac{[1 - \sigma(1 - \eta)]}{q_k} \tilde{u}_k^\alpha \tilde{u}_g^{1-\alpha} \tilde{z}^{1-\alpha} + (\tilde{q}_k - 1)^2 \frac{2}{h_1 \tilde{q}_k} - \delta_k(\tilde{u}_k) = (1 - \tau_b)\tilde{r} \]

Equations (23a)-(23c) can be solved for \( \tilde{z}, \tilde{q}_k, \) and \( \tilde{r} \). Given this solution, \( \tilde{c} \) can be determined from the economy’s aggregate resource constraint:

\[ (1 - g)A\tilde{u}_k^\alpha \tilde{u}_g^{1-\alpha} \tilde{z}^{1-\alpha} = \tilde{c} + \frac{\tilde{q}_k^2 - 1}{2h_1} + \frac{h_2}{2} \frac{[g(A\tilde{u}_k^\alpha \tilde{u}_g^{1-\alpha} \tilde{z}^{1-\alpha})]^2}{\tilde{z}} \]  

4.2 Optimal Fiscal Policy

Under government provision of infrastructure, the burden of replicating the central planner’s allocation falls entirely on the tax system. Comparing (23a)-(23c) with (16a)-(16c), we see that the optimal tax on bond income must be zero as before, i.e., \( \tau_b = 0 \). This implies that the income tax rate must correct for the two externalities in the steady-state equilibrium:

\[ \tilde{\tau}_y = \frac{\varepsilon + (1 - \eta)(1 - \sigma) - (\eta - \varepsilon) \tilde{v}g}{1 - \sigma(1 - \eta)} \]  

Comparing (24) with (17a), its counterpart in the privatized economy, we see that the steady-state optimal tax on income differs across the two regimes of infrastructure provision. This happens because under direct government provision of infrastructure, the allocation of output to infrastructure investment, \( g \), is arbitrary and therefore may be above, below, or equal to its social optimum, \( \tilde{g} \). Hence the term \( (\eta - \varepsilon) \tilde{v}g \) in (24) corrects for this deviation, with \( \tilde{v} \) denoting the shadow value of allocating an extra unit of output to infrastructure investment, as described in section 3.2. Therefore, when \( g < \tilde{g} \), we must have \( \tilde{v} > 0 \), and the optimal tax rate is smaller than in the privatized economy. This encourages private capital accumulation which, by increasing the flow of output, increases the stock of infrastructure towards its socially optimal level. On the other hand, when \( g > \tilde{g} \), we must have \( \tilde{v} < 0 \), as the stock of infrastructure is too large relative to the social optimum. As a result, the optimal tax rate is larger than in the privatized economy. Finally, when \( g = \tilde{g} \), infrastructure investment is at its social optimum and \( \tilde{v} = 0 \). In this case, the optimal income tax, in the steady state, is exactly identical across regimes.

\[ ^{13} \text{The linearized dynamics corresponding to this steady state can be described as } \dot{J} = \Pi (J - J'), \text{ where } J' = (z, q_k), \dot{J} = (\tilde{z}, \tilde{q}_k), \text{ and } \Pi \text{ represents the } 2 \times 2 \text{ coefficient matrix of the linearized system. The equilibrium is characterized by a saddle path with one stable and one unstable eigenvalue.} \]
However, one important caveat to this result distinguishes itself from the design of optimal fiscal policy in the privatized economy. If the government sets $\hat{\tau}_y$ according to (24), the adjustment path followed by the decentralized economy will fail to replicate that of its centrally planned counterpart. This happens because when the government provides the entire stock of infrastructure, the private agent takes it as exogenously given and therefore does not internalize (i) the effect of its private investment decisions on the implied shadow price of infrastructure and (ii) the utilization and depreciation rates of infrastructure capital and their consequences for the corresponding choices for private capital, along the transition path. This behavioral aspect leads to an externality in private resource allocation in the transition to the steady-state equilibrium. As a result, a constant income tax rate cannot account for this transitional externality, and the economy converges to the first-best steady-state equilibrium at a non-optimal rate relative to the centrally planned economy. Therefore, $\hat{\tau}_y$ is the first-best tax rate only in the steady state, but not in transition.

This problem, however, can be corrected by a time-varying income tax rate, which takes the following form:

$$\tau_y(t) = \hat{\tau}_y + \theta [z(t) - \hat{z}]$$  \hspace{1cm} (25)

The income tax rate in (25) tracks the dynamic evolution of the economy as the ratio of infrastructure to private capital changes in transition, thereby enabling the private agent to track the dynamic adjustment of the shadow price of infrastructure as well as its utilization rate.$^{14}$

Comparing (17a)-(17c) with (24) and (25), we see that the mode of infrastructure provision (private or public) generates fundamental differences with respect to the design of optimal fiscal policy. While the optimal income tax and expenditure (subsidy) rates in a privatized economy are constant throughout transition, the optimal income tax rate under government provision must be time-varying until it reaches the steady-state equilibrium. These differences highlight the subtle, but crucial differences between the two regimes regarding the degree to which the various interdependencies between private capital and infrastructure are internalized by the private sector along the transition path to the long-run equilibrium.

5 Private versus Government Provision: A Numerical Analysis

We begin by numerically characterizing the benchmark steady-state equilibrium, where there are no congestion or production externalities, i.e., $\sigma = 1$ and $\varepsilon = 0$. Our starting point is the laissez-faire economy where both private capital and infrastructure are privately provided. Given

$^{14}$The accurate determination of the constant $\theta$ is crucial for the first-best tax policy to replicate the dynamic adjustment of a centrally planned economy. To ensure this, the government must set $\theta$ such that $F(\mu, \theta) = V(\mu) = 0$, where $F(.)$ and $V(.)$ are polynomials derived from the determinants of the linearized matrix of coefficients in the centrally planned and decentralized economies, respectively, while $\mu$ is the stable eigenvalue in the centrally planned economy. When $\theta$ is chosen in this way, the speed of adjustment in the decentralized economy will replicate that of the centrally planned economy. Moreover, $\theta$ is only relevant along the transition path, and does not affect the steady-state equilibrium. As $z(t) \rightarrow \hat{z}$, $\tau_y(t)$ will converge to its long-run optimal rate, $\hat{\tau}_y$. For a more elaborate proof, albeit in a different context, see Turnovsky (1997).
the allocation under laissez-faire, we calibrate an economy where infrastructure is provided by the government, which the private sector takes as exogenously given in making its own allocation decisions. The structural and policy parameters we choose for our calibration are outlined below:

<table>
<thead>
<tr>
<th>Preference Parameters:</th>
<th>$\gamma = -1.5$, $\beta = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Parameters:</td>
<td>$A = 1.5$, $\eta = 0.8$, $h_1 = h_2 = 15$, $\phi_k = \phi_g = 2$</td>
</tr>
<tr>
<td>Externality Parameters:</td>
<td>$\sigma \in [0, 1]$, $\varepsilon \in [0, 0.2]$</td>
</tr>
<tr>
<td>Policy Parameters:</td>
<td>$\tau_y = 0$, $\tau_b = 0$, $s = 0$</td>
</tr>
</tbody>
</table>

The structural parameters have been chosen to be consistent with their corresponding empirical estimates. For example, the preference parameters $\beta$ and $\gamma$ yield an intertemporal elasticity of substitution in consumption equal to 0.4, consistent with the findings of Ogaki and Reinhart (1998). The output elasticity of private capital ($\eta$) is set at 0.8, which is reasonable when we define private capital as an amalgam of physical and human capital, as in Romer (1986). This also implies that the corresponding elasticity for infrastructure is 0.2, which is within the empirically estimated range of 0.1 – 0.3; see Gramlich (1994). The adjustment cost parameters are consistent with Ortiguera and Santos (1997) and their equality serves as a plausible benchmark. While $A$ represents a scale parameter in the production function, the choice of $\phi_k$, the elasticity of depreciation with respect to private capital utilization is set at 2, following Basu and Kimball (1997). Since there is no known estimate for the corresponding elasticity with respect to infrastructure capital, $\phi_g$, we set it equal to $\phi_k$. We vary the congestion parameter ($\sigma$), from 0 (proportional congestion) to 1 (no congestion), while the production externality ($\varepsilon$) is varied from 0 to 0.2, as in Chatterjee and Turnovsky (2006). We set the tax and subsidy rates to zero in the laissez-faire economy, while under government provision we assume that the necessary public expenditure on infrastructure is financed by appropriately adjusting lump-sum taxes or government debt.

In comparing the two regimes of infrastructure provision, we must start from a common benchmark equilibrium across the two regimes. To achieve this outcome, we note that in the government provision regime, (i) spending on infrastructure ($g$), represents an arbitrary policy choice, and (ii) the utilization and depreciation rates for infrastructure capital ($\bar{u}_g$ and $\bar{\delta}_g$) are exogenous constants. Therefore, we calibrate these variables in the government provision economy to equal the corresponding equilibrium values in the private provision (laissez-faire) regime. Table 1A, therefore, depicts the common benchmark equilibrium in the laissez-faire and government provision economies. Since there are no externalities in this equilibrium ($\sigma = 1$ and $\varepsilon = 0$), and the income tax and subsidy parameters are set to zero, this outcome can be viewed as the "first-best." The equilibrium ratio of infrastructure to private capital ($z$) is 0.25, while the shadow prices of infrastructure

\footnote{There have been a few attempts in the literature to measure the elasticity parameter $\phi$, and the ones available show significant variation. For example, Burnside and Eichenbaum (1996) estimate $\phi_k = 1.56$ for U.S. manufacturing, while Finn’s (1995) estimate is 1.44. More recently, Dalgaard (2003) calculates $\phi_k$ to be about 1.7 for Denmark. Basu and Kimball (1997) note that the upper bound of the 95 percent confidence interval for $\phi_k$ is 2.}
and private capital are equalized at 2.42. The steady-state interest rate is 9.85 percent, while the consumption-private capital ratio \( c \) is 0.23. The equilibrium utilization and depreciation rates for private capital and infrastructure in the laissez-faire economy are also equal at 0.38 and 7 percent, respectively. The corresponding variables in the government provision economy are calibrated to equal those in the laissez-faire economy. Table 1B reports the equilibrium fractions of output devoted to consumption, private capital and infrastructure, as well as the steady-state growth rate and relative welfare levels in the two regimes. The consumption-output ratio is 0.53, while the allocation to private investment is about 22 percent. This, in turn, leads to a private capital-output ratio of 2.34. In the laissez faire economy, the private agent allocates 5.5 percent of output to infrastructure investment in equilibrium. The corresponding expenditure in the government provision economy is an arbitrary policy choice and is therefore set to equal the allocation in the laissez faire economy. These equilibrium allocations imply that both economies grow at the common rate of 2.34 percent in the long run and have exactly the same level of welfare. The coincidence of the long-run equilibria in the two models of infrastructure provision provides us with a convenient starting point for analyzing the relationship between fiscal policy and macroeconomic performance in the two regimes.

Table 2 shows the impact of the two externalities (\( \sigma \) and \( \varepsilon \)) on equilibrium growth and welfare. For the purpose of comparison, we report the results relative to the equilibrium in the benchmark economy (where \( \sigma = 1 \) and \( \varepsilon = 0 \)). For example, when \( \sigma = 0.5 \) and \( \varepsilon = 0.1 \), the equilibrium growth rate (identical in the two regimes) is 9 percent below the benchmark level (\( \Psi / \tilde{\Psi} = 0.91 \)) and welfare is 38 percent below the benchmark level (\( W / \tilde{W} = 0.62 \)). From Table 2, we see that for any given value of the production externality (\( \varepsilon \)), an increase in congestion (\( \sigma \) decreases from 1 toward 0), raises equilibrium growth and lowers welfare relative to the benchmark equilibrium. This happens because with higher congestion, the return to private investment increases above its social optimum for any given stock of infrastructure, by increasing the underlying services derived from it. This leads to higher private investment and growth relative to the benchmark. The higher investment implies that fewer resources are devoted to consumption, leading to lower equilibrium welfare. On the other hand, for a given level of congestion, an increase in \( \varepsilon \) lowers the impact of the aggregate production externality, given by \( \tilde{z}^e \), since \( 0 < \tilde{z} < 1 \) and \( 0 < \varepsilon < 1 \). Since \( z = K_g / K \), the aggregate productivity benefit from infrastructure falls relative to private capital, causing the private agent to shift more resources into private investment. Since the stock of private capital was larger than the stock of infrastructure to begin with (\( \tilde{z} < 1 \)), this substitution is subject to

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16 Note that in the government provision economy, the shadow price of infrastructure, \( q_g \), does not apply, since the private agent takes the government-provided stock as exogenously given.

17 Welfare is calculated by numerically integrating the intertemporal utility function (1), where \( C(t) \) is evaluated along its equilibrium path.

18 Since we have set our benchmark tax and subsidy rates to zero, our results should be viewed as a numerical illustration of the analytical framework developed in sections 2-4, rather than a calibration for a particular economy. However, most of the equilibrium values lie within their corresponding ranges for the OECD countries, as reviewed by Morshed and Turnovsky (2004).

19 The equilibrium magnitude of \( z \) is invariant to changes in \( \varepsilon \), because the private returns to both infrastructure and private capital are always equal in equilibrium, and \( \varepsilon \) is external to the agent’s allocation problem.
diminishing returns, which lowers both growth and welfare relative to the benchmark equilibrium.

5.1 Fiscal Policy, Externalities, and Transitional Dynamics

This section considers the effects of fiscal policies on equilibrium allocation and welfare in the two regimes of infrastructure provision. In particular, we focus on the two fiscal instruments that have a direct impact on the incentives to invest: the infrastructure subsidy ($s$) and the income tax rate ($\tau_y$).\textsuperscript{20} We compare the effects of these instruments numerically by considering three policy experiments: (i) a targeted infrastructure subsidy to private providers versus an equivalent increase in public investment under government provision, both financed by lump-sum taxation, (ii) experiment (i), but when spending is financed by a distortionary income tax, and (iii) an increase in the income tax rate. The corresponding results are reported in Tables 3-4 and figure 1.\textsuperscript{21} As we will see in the subsequent sections, though the two regimes start with identical equilibrium allocations, fiscal policy shocks lead to distinct differences in their responses, which ultimately generate substantial differences in long-run welfare levels.

5.1.1 Infrastructure Subsidy versus Government Investment

Tables 3A and 3B report the long-run response to an increase in the infrastructure subsidy rate in the private provision regime with an increase in public spending on infrastructure in the government provision regime. To compare the responses of the two regimes to these policy shocks, we calibrate the increase in public investment under government provision to equal the increase in infrastructure investment under private provision (generated by the subsidy). The difference between Tables 3A and 3B is in the mode of financing the underlying investment in infrastructure: in Table 3A, the spending is financed by a lump-sum tax, while in Table 3B the financing instrument is the distortionary income tax.\textsuperscript{22}

Under lump-sum tax financing (Table 3A), an infrastructure subsidy to private providers drives a wedge between the returns to the two capital stocks by lowering the cost of infrastructure investment and raising its market (shadow) price ($q_g$) relative to that of private capital ($q_k$).\textsuperscript{23} This generates a long-run increase in the allocation of output to infrastructure relative to private capital, reflected by an increase in $G/Y$ and a decline in $I/Y$. The higher return to infrastructure also increases its rate of utilization. Consequently, this raises the productivity of private capital and its rate of

\textsuperscript{20}Since the optimal tax on interest income is zero (from 17b), we set $\bar{r}_b = 0$ throughout our calibration exercises.

\textsuperscript{21}Since both economies have a common pre-shock equilibrium, we report the long-run changes in each variable following an underlying fiscal shock. Therefore, if $x$ is an endogenous variable, we report $dx = x_1 - x_0$, where $x_1$ is the after-shock steady-state equilibrium value of $x$ and $x_0$ is its pre-shock level. The changes in the growth rates are reported as percentages.

\textsuperscript{22}We introduce a comparison between lump-sum and distortionary tax-financing of the subsidy and government investment to relate our results to the previous literature, namely Devarajan et al. (1998) and Chatterjee (2006), who consider only the effects of a distortionary financing instrument (income tax).

\textsuperscript{23}Note here that since the infrastructure subsidy is tied to the cost of public investment (and not output), we calibrate $s = 0.5$ so that a subsidy that finances 50 percent of the cost of infrastructure investment equals about 10 percent of GDP.
utilization as well. The increase in the rates of accumulation and utilization of the two capital stocks raises the equilibrium growth rate and increases the flow of output in a proportion larger than consumption (reflected by a decline in the consumption-output ratio).

Under government provision, however, an equivalent increase in public spending \((g)\) raises the total cost of infrastructure investment, since the accumulation of infrastructure is subject to installation costs that must also be financed by the government. As a result, there is a larger crowding out of private investment and consumption relative to the privatized economy, as the government uses the economy’s resources in accumulating and installing infrastructure. In fact, the fall in the consumption-output ratio is indeed very large relative to that in the privatized economy. On the other hand, the large amount of resources devoted by the government to infrastructure (new investment and installation costs) raises the growth rate more than that under private provision.

When the mode of financing is changed to the distortionary income tax (Table 3B), the above mechanisms remain qualitatively the same. However, since the income tax used to finance the subsidy under private provision lowers the after-tax marginal return on both types of capital, the crowding out of private investment is larger and the increase in infrastructure investment smaller than under lump-sum tax financing. The lower after-tax marginal return on the two types of capital causes a large substitution in favor of consumption, which leads to an increase in the consumption-output ratio. Since the higher income tax is being used to finance infrastructure investment, it causes modest increases in the rates of utilization and long-run growth. Under government provision, since infrastructure is publicly provided, the extent of crowding out of private investment is smaller, thereby leading to a more expansionary effect on the growth rate than under private provision.

**Transitional Dynamics**

The dynamic adjustment in the two regimes in response to a subsidy and an increase in government investment is illustrated in figure 1(panels A and B, respectively).

The transitional dynamics in the two economies are dramatically different, although their long-run responses are qualitatively similar. In the private provision model, an increase in the subsidy to infrastructure generates a huge increase in expected long-run productivity. However, since the stock of infrastructure capital cannot be changed instantaneously, the agent responds by increasing the rate of its utilization, \(u_g\). As a result, \(u_g\) jumps up instantaneously to overshoot its higher long-run equilibrium (fig. 1Ai). The higher expected long-run stock of infrastructure also raises the expected long-run productivity of private capital, causing its rate of utilization, \(u_k\), to jump up as well, but by less than the jump in \(u_g\), to maintain equality in their respective rates of return. Thereafter, as infrastructure is accumulated, its average product falls and \(u_g\) gradually declines and approaches its new steady-state equilibrium rate from above. At the same time, the rising stock of infrastructure raises the productivity of private capital, and \(u_k\) increases in transition to approach its new (common) equilibrium rate from below. In the new steady-state equilibrium, both capital
stocks have the same rates of utilization. In contrast, in the government provision model, the private agent cannot alter the utilization rate of infrastructure, since it is exogenous to private decisions. As a result, \( u_g \) in the government provision model remains unchanged throughout transition. The higher government spending therefore leads to a slight downward jump in the utilization rate for private capital (fig. 1Bi). Thereafter, as infrastructure is accumulated, the positive productivity effect on private capital raises its utilization rate gradually to its new, higher steady-state equilibrium.

Figures 1A and B (ii) illustrate the dynamic response of the private investment-output ratio in the two regimes. In the private provision model, the increase in infrastructure investment due to the subsidy requires a substitution away from private capital investment, leading to an instantaneous decline in \( I/Y \). The large investment boom causes output to grow at a much faster rate than private capital in transition, causing the private investment-output ratio to gradually decline to its lower after-shock equilibrium level. In the government provision model, the appropriation of private resources for infrastructure investment by the government (due to installation costs) causes the agent to increase its allocation to private investment to maintain the flow of output. This causes an initial upward jump in \( I/Y \). Thereafter, as public capital accumulates, output grows much faster than private capital, and the investment-output ratio falls sharply to its lower steady-state equilibrium.

The responses of the consumption-private capital and consumption-output ratios are depicted in figures 1A and B (iii)-(iv). Again, we see that the dynamic responses across the two regimes are dramatically different. In the private provision economy, the higher long-run productivity of private capital (due to infrastructure accumulation and higher utilization) causes the consumption-capital and the consumption-output ratios to increase instantaneously. Thereafter, the investment boom causes output to increase at a rate higher than consumption, so that the consumption-output ratio falls in transition. On the other hand, the large increase in output permits the growth rate of consumption to exceed that of private capital, so that the consumption-capital ratio increases in transition. In the government provision model, consumption is crowded out instantaneously as the agent tries to offset the higher spending by the government by substituting away from consumption into private investment. This is shown by the large downward jumps of the consumption-capital and consumption-output ratios. Thereafter, the growth in output and private capital (due to infrastructure investment by the government) causes these ratios to increase over time, indicating that the benefits of investment and growth are reallocated somewhat back to consumption during transition to make up for the large initial decline.

5.1.2 An Income Tax Increase

Table 3C reports the long-run effects of a 10 percent increase in the income tax rate across the two regimes, from its benchmark rate of \( \tau_y = 0 \) to 0.1. In general, an increase in the tax on income will reduce the after-tax returns on both private capital and infrastructure in the privatized
economy. This leads to a substitution away from investment in both types of capital towards consumption. As a result, the consumption-output ratio increases, while the allocations to the two types of investment fall. The allocation to private investment declines more than the one to infrastructure because, given their respective output elasticities, private investment has a larger initial share in output. On the other hand, the lower after-tax marginal product of the two capital stocks causes the private agent to lower the respective utilization rates \(u_k\) and \(u_g\) which, by reducing depreciation costs, partially offsets the decline in investment. Overall, the decumulation of the two capital stocks and their lower utilization lead to a decline in the long-run equilibrium growth rate.

In the economy where the government provides the entire stock of infrastructure, the private agent cannot alter investment in infrastructure or its utilization. Since these variables are exogenous to the private agent, the burden of adjustment falls entirely on the stock of private capital. As a result, the decline in the private investment-output ratio is larger than in the privatized economy. However, since the government maintains its rate of investment in infrastructure (unlike the privatized economy, where it declines), the decline in productivity of private capital is less than that under private provision. Consequently, the reduction in the rate of utilization of private capital is also less than that in the privatized economy. This implies a smaller substitution towards consumption, and consequently, a smaller decline in the equilibrium growth rate relative to the privatized economy.

**Transitional Dynamics (or the Lack of It)**

In contrast to the effects of tax shocks in models with fixed depreciation and capital utilization rates (where typically \(u_k = u_g = 1\)), there is little or no dynamic adjustment when these variables are endogenous. For example, in the private provision regime, the income tax shock does not generate any dynamic adjustment and the economy immediately jumps to its after-shock steady-state equilibrium. This is a surprising result, since traditionally models with multiple capital stocks display slow adjustment to tax shocks; see Futagami et al. (1993). However, when one considers the role of capital utilization and depreciation in resource allocation, this result is not difficult to rationalize. Since the utilization decisions provide the agent an extra margin along which the flow of output can be maintained, the ratio of public to private capital, \(z\), remains invariant to a tax increase. This happens because, on the margin, the tax shock affects the returns to either type of capital symmetrically. Also, the no-arbitrage conditions (9g) and (9h) imply that the after-tax equality in the return to both types of capital must be maintained at all points of time. This is ensured as the agent, in response to the tax increase, reallocates resources away from investment in the two capital goods and readjusts their respective utilization rates to maintain \(z\) at a constant level. Therefore, although the equilibrium allocations and the growth rate change, the effect of the tax increase on the economy is instantaneous. In the government provision model, since one adjustment margin is not available to the agent (infrastructure investment and its utilization), a tax shock does generate a dynamic response, but the adjustment is very quick as the
agent appropriately adjusts the utilization rate of private capital. Therefore, we have chosen only to discuss the qualitative nature of the dynamics, but not illustrate them graphically.

5.1.3 Welfare Effects of Fiscal Shocks and the Role of Capital Utilization

One crucial norm for comparing the relative performance of the two regimes of infrastructure provision is the response of economic welfare to underlying changes in fiscal instruments. It is also instructive to check the robustness of the welfare responses to variations in the externality parameters. We conduct these exercises for the subsidy and government spending increase in Tables 4A and 4B, and the income tax increase in Table 4C.

The first thing to note about Tables 4A-B is that a targeted subsidy to private providers of infrastructure yields uniformly higher welfare gains than an equivalent increase in government investment, irrespective of whether the financing instrument is a lump-sum tax or an income tax. This comparison is also robust to changes in either the congestion or production externality parameters. This is a significant result, since it has been generally thought that if the government has access to lump-sum taxes, the choice between private and public provision of infrastructure is irrelevant (in terms of the impact on welfare); see Devarajan et al. (1998). However, our results indicate that even with lump-sum taxes and debt-financing at its disposal, the government cannot outperform the private sector in the context of infrastructure provision. Much of the earlier analysis of this issue has relied heavily on the asymmetric distortions created by the income tax in financing infrastructure investment in the two regimes. Our results indicate that one must look beyond tax distortions to explain the inherent differences between the two modes of infrastructure provision. The intuition can instead be drawn from the behavioral differences between the two regimes, such as the asymmetric response of capital utilization decisions to the underlying fiscal policy shocks.

Under private provision, the subsidy lowers the cost of infrastructure investment and increases the rates of utilization of both the capital stocks. The resultant flow of output is therefore much higher than under government provision, where the private agent cannot alter the rate of infrastructure utilization to its advantage. Moreover, an increase in direct government spending on infrastructure increases installation costs, which leads to a larger crowding out of private consumption under government provision. On the other hand, the ability to control the allocation of resources to the two capital stocks by adjusting their rates of utilization requires a smaller adjustment in consumption in the privatized economy. These behavioral responses lead to dramatic differences in welfare levels across the two regimes. For example, in Table 4A, when there are no externalities ($\sigma = 1$ and $\varepsilon = 0$), the welfare gain from a lump-sum tax-financed subsidy to private providers is 29.57 percent, while that from an equivalent increase in government investment is only 1.52 percent. In fact, when $\varepsilon = 0$, an increase in congestion leads to increasing welfare losses under government provision, while there are increasing gains under private provision. This happens because, when congestion is higher, the substitution away from consumption under government
provision is also higher, as the agent accumulates more private capital in order to increase the services from the stock of infrastructure. Therefore, infrastructure accumulation under government provision makes the effects of congestion worse, reducing welfare in the long run. In contrast, a subsidy in the private provision model causes a smaller substitution away from consumption into private investment, as the agent can adjust both the capital utilization rates to derive the necessary services from infrastructure. This lowers the adverse effects of congestion and makes the economy better off. Since a subsidy increases the stock of infrastructure relative to private capital ($z$), an increase in $\varepsilon$ now has a larger productivity impact on the economy, and the welfare gains from infrastructure investment in both regimes increase, although the privatized economy still yields much higher welfare gains than the government provision economy.

When the two economies are subject to an income tax shock (Table 4C), the comparisons in Tables 4A and 4B are now reversed: the economy under government provision yields uniformly higher welfare gains or lower welfare losses relative to the privatized economy. This happens because a tax shock is more contractionary for the privatized economy. By reducing the after-tax return on both private capital and infrastructure, a tax shock leads not only to lower aggregate investment in the privatized economy, but also to lower utilization rates for both capital stocks. As a result, the flow of output declines, yielding only modest welfare changes. In comparison, a tax increase under government provision reduces only the return and utilization of private capital. Moreover, since the government maintains its infrastructure investment rate, the resultant fall in output is partially offset. This leads to higher welfare gains or lower losses than under private provision. For example, when $\sigma = 1$ and $\varepsilon = 0$ in Table 4C, a 10 percent increase in the income tax rate lowers welfare by 0.88 percent in the privatized economy, while the corresponding loss under government provision is 0.53 percent. Therefore, an income tax is more distortionary in the private provision model than under government provision. As the level of congestion increases, a tax increase under government provision has a larger positive impact than under private provision, since the government, by maintaining its expenditure on infrastructure, ensures that the corresponding services derived by the private agent are unaffected. An increase in the production externality increases the welfare gains in both regimes, but again, the government provision model performs better. Interestingly, this result is exactly the opposite to that of Devarajan et al. (1998), where tax distortions are higher under government provision. Endogenous utilization decisions, therefore, serve as the crucial link between our results and those in the literature.

The welfare comparisons provided in Table 4 suggest that the desirability of private and government provision regimes depends not on structural parameters or distortions created by the tax system (lump-sum versus distortionary), but rather on the private sector’s ability to internalize capital utilization rates in response to fiscal shocks, and its consequences for resource allocation.
6 Conclusions

Though infrastructure provision has recently become a contentious issue in policy circles, it has received very little formal attention from economists. This paper builds on a very small but promising literature and attempts to provide some insights into the choice between private and government provision and its impact on an economy’s aggregate performance. We distinguish our work from the existing literature by focussing on the behavioral aspects of excludability (or the lack of it) in the accumulation and usage of infrastructure. Specifically, by introducing both private and infrastructure capital utilization as decision variables, we have illustrated how the mode of infrastructure provision affects the degree to which these decisions are internalized and how, in turn, they affect an economy’s response to fiscal shocks. We also show that the choice between private and public provision matters even when the government has access to lump-sum taxes or other non-distortionary fiscal instruments. If the government wants to stimulate investment in infrastructure, then a subsidy to private providers yields significantly higher welfare gains than an equivalent increase in direct government investment, irrespective of the mode of financing. By contrast, the effects of an income tax are more distortionary under private provision than under government provision. Our comparisons are robust to different aspects of rivalry in infrastructure provision, like congestion and aggregate production externalities. The robustness of our results to financing instruments and structural parameters represents a significant departure from previous work in this area.

The mode of infrastructure provision also has an important bearing on the design of optimal fiscal policy. While under private provision, both an (constant) income tax and an infrastructure subsidy are jointly required to attain the first-best equilibrium, under government provision the burden of attaining optimality falls on the income tax alone, which in turn needs to be time-varying. Since the private agent treats the underlying stock of infrastructure as exogenous, it fails to internalize the effect of private capital accumulation and usage on the implied utilization rate and shadow price of infrastructure. A time-varying income tax rate is thus required to track these variables for the agent. Therefore, the mode of infrastructure provision and the consequent behavioral differences play a critical role in determining the relationship between fiscal policy and macroeconomic performance in a growing economy.
### TABLE 1
Benchmark Equilibrium in the Laissez-faire and Government Provision Models
\((\sigma = 1, \varepsilon = 0)\)

1A. Equilibrium Variables

<table>
<thead>
<tr>
<th>(z)</th>
<th>(q_j)</th>
<th>(r) (%)</th>
<th>(c)</th>
<th>(u_j)</th>
<th>(\delta_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>2.42</td>
<td>9.85</td>
<td>0.23</td>
<td>0.38</td>
<td>0.07</td>
</tr>
</tbody>
</table>

1B. Equilibrium Ratios, Growth, and Welfare

<table>
<thead>
<tr>
<th>(C/Y)</th>
<th>(I/Y)</th>
<th>(K/Y)</th>
<th>(G/Y)</th>
<th>(\Psi) (%)</th>
<th>(W_i/W_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.53</td>
<td>0.22</td>
<td>2.34</td>
<td>0.055</td>
<td>2.34</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** (a) \(j = k, g\) and (b) \(i = G\) (Government Provision), \(L\) (Laissez-faire/Private Provision)

### TABLE 2
Sensitivity of Equilibrium Growth and Welfare to Externalities

<table>
<thead>
<tr>
<th>(\sigma = 0)</th>
<th>(\sigma = 0.5)</th>
<th>(\sigma = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Psi/\bar{\Psi})</td>
<td>(W/W)</td>
<td>(\Psi/\bar{\Psi})</td>
</tr>
<tr>
<td>(\varepsilon = 0)</td>
<td>1.32</td>
<td>0.88</td>
</tr>
<tr>
<td>(\varepsilon = 0.1)</td>
<td>1.03</td>
<td>0.56</td>
</tr>
<tr>
<td>(\varepsilon = 0.2)</td>
<td>0.78</td>
<td>0.36</td>
</tr>
</tbody>
</table>

**Note:** \(\Psi/\bar{\Psi}\) = Equilibrium growth rate relative to benchmark growth rate \((\sigma = 1, \varepsilon = 0)\)
\(W/W\) = Equilibrium welfare relative to benchmark welfare \((\sigma = 1, \varepsilon = 0)\)


**TABLE 3**

Private versus Government Provision of Infrastructure: Steady-State Effects of Fiscal Shocks  
\(\sigma = 1, \varepsilon = 0\)

A. Subsidy to Private Providers versus Government Spending (Lump-sum Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>(d(C/Y))</th>
<th>(d(I/Y))</th>
<th>(d(G/Y))</th>
<th>(du_k)</th>
<th>(du_g)</th>
<th>(d\Psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Provision Model</td>
<td>-0.18</td>
<td>-1.20</td>
<td>+4.91</td>
<td>+2.14</td>
<td>+2.14</td>
<td>+0.57</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>-8.99</td>
<td>-1.24</td>
<td>+4.91</td>
<td>+2.23</td>
<td>–</td>
<td>+0.59</td>
</tr>
</tbody>
</table>

B. Subsidy to Private Providers versus Government Spending (Income Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>(d(C/Y))</th>
<th>(d(I/Y))</th>
<th>(d(G/Y))</th>
<th>(du_k)</th>
<th>(du_g)</th>
<th>(d\Psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Provision Model</td>
<td>+3.93</td>
<td>-2.26</td>
<td>+4.38</td>
<td>+0.76</td>
<td>+0.76</td>
<td>+0.20</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>-6.14</td>
<td>-1.67</td>
<td>+4.38</td>
<td>+1.36</td>
<td>–</td>
<td>+0.35</td>
</tr>
</tbody>
</table>

C. Income Tax Increase

<table>
<thead>
<tr>
<th></th>
<th>(d(C/Y))</th>
<th>(d(I/Y))</th>
<th>(d(G/Y))</th>
<th>(du_k)</th>
<th>(du_g)</th>
<th>(d\Psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Provision Model</td>
<td>+4.90</td>
<td>-1.41</td>
<td>-0.35</td>
<td>-1.58</td>
<td>-1.58</td>
<td>-0.39</td>
</tr>
<tr>
<td>Government Provision Model</td>
<td>+4.05</td>
<td>-1.48</td>
<td>–</td>
<td>-1.44</td>
<td>–</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

**Note:** Tables 3A-C report the long-run changes following an underlying fiscal shock. For example, if \(x\) is an endogenous variable, we report \(dx = x_1 - x_0\), where \(x_1\) is the after-shock steady-state equilibrium value of \(x\) and \(x_0\) is its pre-shock level. The changes in the growth rates are in percentages.
TABLE 4
Comparison of Welfare Gains/Losses from Policy Interventions

A. Subsidy to Private Providers versus Government Spending (Lump sum Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>( \sigma = 0 )</th>
<th>( \sigma = 0.5 )</th>
<th>( \sigma = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon = 0 )</td>
<td>30.35</td>
<td>5.34</td>
<td>29.92</td>
</tr>
<tr>
<td>( \varepsilon = 0.1 )</td>
<td>49.83</td>
<td>11.14</td>
<td>49.01</td>
</tr>
<tr>
<td>( \varepsilon = 0.2 )</td>
<td>72.14</td>
<td>32.43</td>
<td>70.85</td>
</tr>
</tbody>
</table>

B. Subsidy to Private Providers versus Government Spending (Income Tax-Financed )

<table>
<thead>
<tr>
<th></th>
<th>( \sigma = 0 )</th>
<th>( \sigma = 0.5 )</th>
<th>( \sigma = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon = 0 )</td>
<td>33.55</td>
<td>0.73</td>
<td>30.87</td>
</tr>
<tr>
<td>( \varepsilon = 0.1 )</td>
<td>53.20</td>
<td>16.66</td>
<td>49.86</td>
</tr>
<tr>
<td>( \varepsilon = 0.2 )</td>
<td>75.63</td>
<td>36.89</td>
<td>71.53</td>
</tr>
</tbody>
</table>

C. Income Tax Increase

<table>
<thead>
<tr>
<th></th>
<th>( \sigma = 0 )</th>
<th>( \sigma = 0.5 )</th>
<th>( \sigma = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon = 0 )</td>
<td>2.96</td>
<td>3.04</td>
<td>0.86</td>
</tr>
<tr>
<td>( \varepsilon = 0.1 )</td>
<td>2.97</td>
<td>4.18</td>
<td>0.86</td>
</tr>
<tr>
<td>( \varepsilon = 0.2 )</td>
<td>2.98</td>
<td>5.60</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Note:** Welfare changes in Tables 4A-C are reported as percentages.
Figure 1. Infrastructure Subsidy (Private Provision) vs. Public Spending (Government Provision)

A. Private Provision Model

B. Government Provision Model

i. Capital Utilization Rates

ii. Private Investment-Output Ratio

iii. Consumption-Private Capital Ratio

iv. Consumption-Output Ratio
References


